TEKSING TOWARD STAAR

MATHEMATICS®

GRADE 8

Lesson Projection Masters

Six Weeks 1
Lesson 1

Understanding Real Numbers

A group of items or numbers is called a set. A part of that set is called a **subset**. The set of numbers we use in our every day lives is the set of real numbers. These are the numbers that are located on a number line. One subset of the real numbers is the set of whole numbers. **Whole numbers** are the numbers 0, 1, 2, 3, 4… Each of these numbers has an opposite 0, -1, -2, -3, -4… When the whole numbers and their opposites are joined together the set of **integers** is created.

The set of integers are indicated in set notation as {…-4, -3, -2, -1, 0, 1, 2, 3, 4…}.

If zero is removed from the set of whole numbers, the set of **natural numbers** or **counting numbers** is created. The natural numbers can be indicated in set notation as $\{1, 2, 3, 4, 5, 6, 7, ...\}$. The whole numbers, counting numbers, and integers are all subsets of a larger set called the rational numbers. When a number of the form $\frac{a}{b}$ is created where *a* and *b* are both integers but $b \neq 0$, then the set of **rational numbers** is created. For example, the ratio of 3 to 5 creates $\frac{3}{5}$, so $\frac{3}{5}$ is a rational number. The ratio of 20 to 2 creates $\frac{20}{2}$ or 10 which is a whole number and also a rational number.

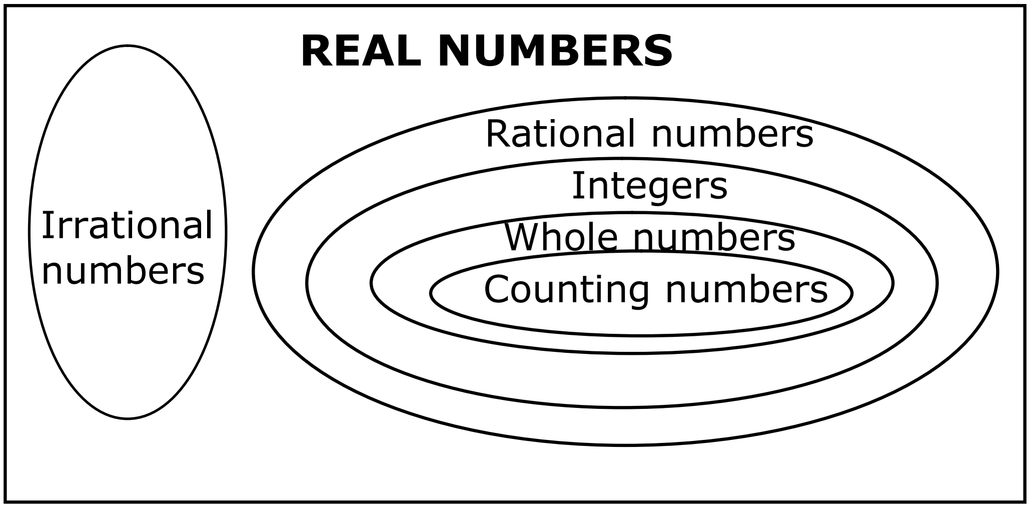
A mixed number like $5\frac{1}{2}$ is a rational number because it can be rewritten as an improper fraction, $\frac{11}{2}$, which is the ratio of two integers.

The real numbers, which are all the numbers on a number line, are divided into 2 large subsets. The **rational numbers** and the **irrational numbers**.

The irrational numbers are made up all numbers that CANNOT be expressed as a ratio of two integers. Some irrational numbers are numbers like $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$ and π. Any decimal that does NOT terminate and does NOT repeat is an irrational numbers. For example, 5.123456… is an irrational decimal.

A repeating decimal has a block of one or more digits that repeats indefinitely. Those decimals, such as 0.3333333…. or 4.1212121212….are rational decimals.

The relationship between these sets of subsets of the real numbers can be shown with a Venn diagram.



This diagram shows that the rational numbers and the irrational number are disjointed. They do NOT share any numbers. The prefix Ir means "not" so irrational means "not rational".

In determining if a radical number such as $\sqrt{a}$ is an irrational number or a rational number, be sure to simplify it first. If the number *a* is a perfect square number, then the radical will simplify into an integer. For example, if $b^2 = a$ and *b* is an integer, then a is a perfect square number and $\sqrt{a} = b$ and $-\sqrt{a} = -b$. $\sqrt{a} = b$ is called the **principal root**.

Example: Is $\sqrt{25}$ an irrational or rational number?

Since $5^2 = 25$, then 25 is a perfect square and $\sqrt{25}$ = 5. 5 is rational number.

Therefore, $\sqrt{25}$ is a rational number.

Example: Is $\sqrt{90}$ an irrational or rational number?

Since 90 is between 81 and 100, then $\sqrt{90}$ is between 9 and 10. Since there are no integers between 9 and 10, $\sqrt{90}$ is a decimal between 9 and 10. Using your calculator, find the decimal approximation the calculator will give you for $\sqrt{90}$. The calculator will give 9.486832981…. depending upon how many decimal places you request.

Therefore, $\sqrt{90}$ is an irrational number that can be approximated with the rational number 9.5 or 9.49.

Problem-Solving Model

Step	Description of Step
1	**Analyze the given information.** • Summarize the problem in your own words. • Describe the main idea of the problem. • Identify information needed to solve the problem.
2	**Formulate a plan or strategy**. • Draw a picture or diagram. • Guess and check. • Find a pattern. • Act it out. • Create or use a chart or table. • Work a simpler problem. • Work backwards. • Make an organized list. • Use logical reasoning. • Brainstorm. • Write a number sentence or an equation
3	**Determine a solution.** • Estimate the solution to the problem. • Solve the problem.
4	**Justify the solution.** • Explain why your solution solves the problem.
5	**Evaluate the process and the reasonableness of your solution.** • Make sure the solution matches the problem. • Solve the problem in a different way.

Problem-Solving Questions

Directions:
- **Work with a partner.**
- **Write your answers on notebook paper.**
- **Answer questions 1-3.**
- **Complete the solution to the problem(s).**
- **Answer questions 4-10.**

1. What is the main idea of this problem?
2. What are the supporting details in this problem?
3. What skills, concepts, and understanding of math vocabulary are needed to be able to answer this problem?
4. Did this problem involve mathematics arising in everyday life, society, or the work place?
5. What is a good problem solving strategy for this problem?
6. Can you explain how you used any math tools, mental math, estimation, or number sense to solve this problem?
7. Did this problem involve using multiple representations (symbols, diagrams, graphs, math language)?
8. Did you use any relationships to solve this problem?
9. How can you justify your solution to the problem?
10. How can you check for reasonableness of your solution to this problem?

Problem-Solving 1

Problem 1: Which of the following statements are true? Use T or NT.

______1. All integers are rational numbers.

______2. Any rational number can be expressed as the ratio of two integers.

______3. If a decimal does not terminate, it is an irrational number.

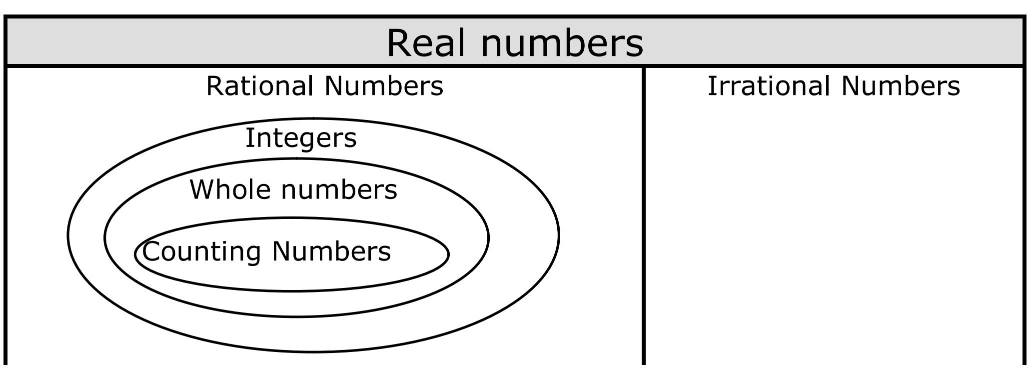
______4. Some integers are irrational numbers.

______5. The set $\{8,\ 8.5,\ \sqrt{10},\ -23\}$ are all rational numbers.

______6. The set $\{-3,\ 19,\ 20,\ 0, -1\}$ are all integers.

For any statement you listed as NT, explain your reasoning.

Problem 2: Place -6, 0, $\sqrt{7}$, $\frac{12}{4}$, $-3\frac{1}{2}$ and $0.\overline{45}$ in the appropriate place on the Venn diagram.



Understanding Number Sets in Real-World Situations

Numbers used in real-world situations can be rational numbers or irrational numbers. However, irrational numbers are often approximated with a rational number to do calculations. For example, if the circumference of a circle is 16π inches, a carpenter would approximate it by using a rational substitution for π. Depending on the accuracy needed, he could use 3 or 3.14 or 3.14156.

Example: The number of dollar bills in a person's wallet

Whole numbers This set best describes the number of dollar bills because the person may have 0, 1, 2, 3, … dollar bills in his wallet.

Example: The length of a side of a square with an area of 25 square feet

Whole Number Since the formula for the area of a square is $s^2 = A$, then $s = \sqrt{A}$ (take square root of both sides but use only the positive or principal root since length has to be positive.)

$s = \sqrt{25} = 5$ and 5 is a whole number.

Example: The length of a side of a square with an area of 40 square feet

Irrational number Since the formula for the area of a square is $s^2 = A$, then $s = \sqrt{A}$ (take square root of both sides but use only the positive or principal root since length has to be positive.)
$s = \sqrt{40}$. Since 40 is between the perfect squares 36 and 49, $\sqrt{40}$ will be a decimal between 6 and 7. There is NOT a terminating or repeating decimal that when multiplied times itself will have a product of 40. $\sqrt{40}$ can be **approximated** with a rational number such as 6.3.

If the square root of a whole number is rational, then its prime factorization can be divided into two equal sets.

Example: $\sqrt{144}$ is rational because the prime factorization of 144 is $(3 \times 2 \times 2)(3 \times 2 \times 2)$. Since the two sets are identical, the $\sqrt{144} = 3 \times 2 \times 2 = 12$, which is a whole number and thus rational.

Problem-Solving 2

Problem 1: Identify the set(s) of numbers that best describe the situations below.

- Numbers used in an area code

____

- Score for the home team in soccer

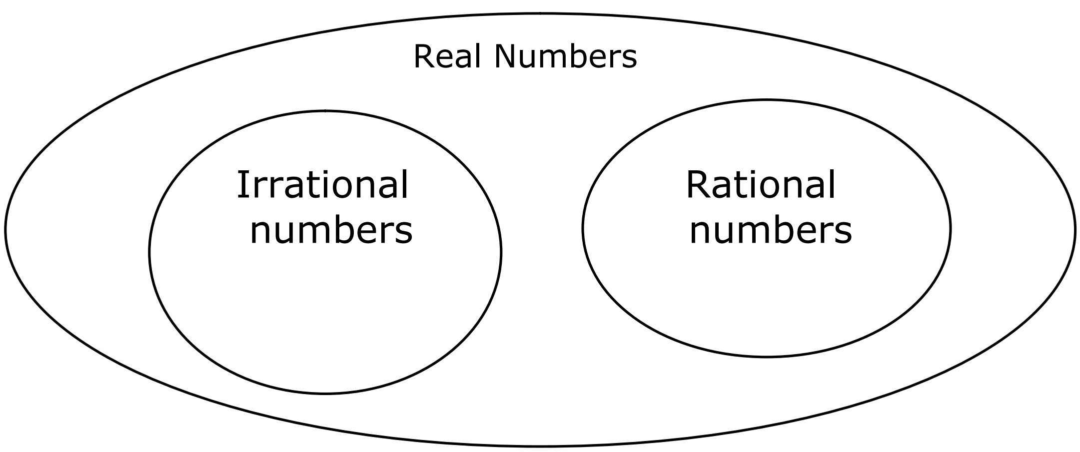
____

- Length of a side of a square with area *A*

____

Problem 2: Place the square root of each counting number 1-18 on the Venn diagram below.

Square Roots of Counting Numbers 1-18



Six Weeks 3
Lesson 8

Determining the Mean Absolute Deviation

A measure of center of data is a single number. One measure of center is the **median** (center data point). Another measure of center of data is the **mean**. The **mean** is the average of the data points. To find the mean, sum the data points and divide by the number of data points. The mean and median do not have to be a data point in the data set.

A measure of variability is a single number that is used to describe the spread of the data. The measure of variability we have already studied is the **range** of the data. Another measure of variability that we will study in this lesson is the **mean absolute deviation**. This value is often referred to as the MAD, which is the mean distance between each value of the data set and the mean of the data set.

The steps to find the mean absolute deviation of a data set are:

1. Find the mean of the data set. (Sum the data points and divide by the number of data points.)

2. Find the distance each data point is from the mean. (A table might be helpful for this step.)

3. Find the mean of these distances. (Use your calculator and round to the nearest tenth.)

A larger mean absolute deviation number tells you that the data is more spread out.

The smaller mean absolute deviation number tells you the values are all closer to the mean of the data set.

An outlier in your set can affect your MAD because it will be farther from the mean than the other data points.

Example: Find the mean deviation number for the set of values below. The values represent the number of blue marbles in various bags of marbles.

Data Set: 6, 8, 5, 2, 7, 7, 4, 6, 7, 8

Find the mean:

$$\frac{6+8+5+2+7+7+4+6+7+8}{10} = \frac{60}{10} = 6$$

Find the distance each data point is from the mean.

Data point	6	8	5	2	7	7	4	6	7	8
Distance from mean	**0**	**2**	**1**	**4**	**1**	**1**	**2**	**0**	**1**	**2**

Find the mean of the distances.

$$\frac{0+2+1+4+1+1+2+0+1+2}{10} = \frac{14}{10} = 1.4$$

The MAD is 1.4. This is a small mean deviation number, but if the data point 2 had been a number closer to 6, the MAD would have been even smaller.

Example: The height of 6 team members of the boys' eighth grade basketball team is given as Data Set 1.
The height of 6 team members of the girls' eighth grade basketball team is given as Data Set 2. The heights are given in inches.

Data Set 1 (boys' heights): 69, 62, 69, 60, 70, 72

Data Set 1 (girls' heights): 57, 60, 67, 63, 61, 62

Compare the mean absolute deviation for the two data sets.

Data Set 1:

Find the mean:

$$\frac{69+62+69+60+70+72}{6}=\frac{402}{6}=67$$

Find the distance each data point is from the mean.

Data point	69	62	69	60	70	72
Distance from	**2**	**5**	**2**	**7**	**3**	**5**

Find the mean of the distances.

$$\frac{2+5+2+7+3+5}{6}=\frac{24}{6}=4$$ **The MAD for the boys' heights is 4 inches.**

Data Set 2:

Find the mean:

$$\frac{57+60+67+63+62+61}{6}=\frac{370}{6}\approx 61.7$$

Find the distance each data point is from the mean.

Data point	57	60	67	63	62	61
Distance from	**4.7**	**1.7**	**5.3**	**1.3**	**0.3**	**0.7**

Find the mean of the distances.

$$\frac{4.7+1.7+5.3+1.3+0.3+0.7}{6}=\frac{14}{6}\approx 2.3$$ **The MAD for the girls' heights is 2.3 inches.**

The MAD for the girls' heights is smaller than the MAD for the boys' heights. The girls' height has less deviation, or variation, in their heights. The heights of these six girls are closer to the mean of the data than the heights of the six boys to the mean of their data.

Problem-Solving 1

Problem 1: What does a small mean absolute deviation tell you about the data set?

Problem 2: How does an outlier affect your mean absolute deviation?

Problem 3: Find the MAD to the nearest tenth for the following data set.

55, 54, 53, 58, 56, 59, 55, 58

Problem 4: If a data set had a MAD of 0, what can you deduce about the data set?

Problem 5: At a cupcake factory, a machine is set to put two-thirds cup of batter in each muffin holder. When checking for accuracy of the amount of batter dispersed by the machine, the person found the MAD to be 0. What does that tell you about the machine?

Mean Absolute Deviations

Group #	Data					Mean of Data	Distances from Mean					Mean Absolute Deviation