

# GRADE 7

## TEKS/STAAR-BASED LESSONS

# Parent Guide

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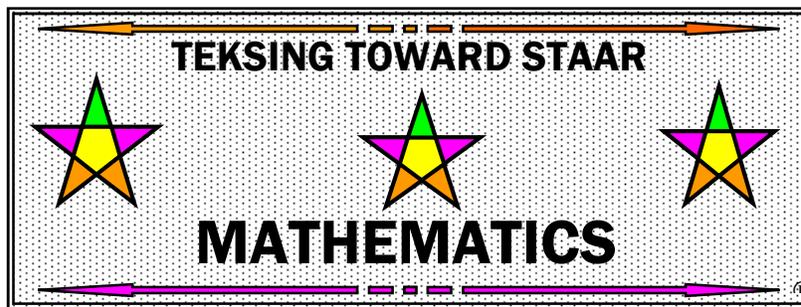
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## OVERVIEW OF THE LESSONS AND THE GOALS

The universal use of calculators and computers has changed what is important in mathematics as well as what students need to know to be prepared for college and the work force. The past focus of mathematics curriculum was to practice and memorize some techniques that are no longer useful because they were isolated from their origins and their uses in the real world.

Current research on how students learn is now telling us that most students cannot learn mathematics effectively and efficiently by being asked to memorize given rules and practicing those rules for mastery of basic math skills. A report to the nation by the National Research Council entitled Everybody Counts stated, "Presentation and repetition may help students do well on some standardized tests and lower order skills, but are generally ineffective for long term learning, for higher-order thinking, and for versatile problem solving."

Recent research has also impacted teaching methods. The research strongly indicates that a teacher telling and/or showing students how to "do math" has very little to do with promoting true learning. Students must construct their own understanding. Research shows that most students learn best when working in partner pairs or small groups to communicate and freely discuss important skills and concepts as they solve problems.

The curriculum is designed to reflect research, to reflect the National Council of Teachers of Mathematics (NCTM) Standards and to meet the requirements of the Texas Essential Knowledge and Skills for grades 6-8 mathematics through focusing on core concepts throughout the year. The intent of this design is to develop students' confidence in their ability to understand and use mathematics as a tool to solve problems as well as help students develop an understanding of the importance of mathematics in relation to their future world.

The curriculum is designed to be composed of many problems – some for spiraled review of skills and concepts already presented, some to help students develop an understanding of new skills and concepts, some to involve the use of hands-on mathematics, some to include other disciplines such as reading, writing, science, social studies, art, and architecture.

The design of each lesson is consistent and includes a format for delivery of instruction, assessment, and homework. Where appropriate, the use of manipulatives and technology is included in the lesson. Cooperative learning as a learning setting is utilized in each lesson.

### ***Use of Manipulatives***

Manipulatives are multisensory tools for learning that provide students with a means of communicating ideas by allowing them to model or represent their ideas concretely. Using manipulatives, however, does not guarantee understanding of a mathematics concept (Baroody, 1989). After allowing students to explore a concept using manipulatives, teachers must formulate questions to elicit the important mathematical ideas that enable students to make connections between the mathematics and the manipulatives used to represent the concepts. The authors of the *TEKSING TOWARD STAAR* Lessons assume that teachers will use manipulatives when appropriate for instruction in their classroom.

### ***Use of Technology***

Developments in technology have made the traditional, computation-dominated mathematics curriculum obsolete. As a result, the authors of this curriculum assume that grade 5 students will have access to appropriate calculators. Also assumed is the use of computers for demonstration purposes as well as cooperative group work or individual work.

### ***Use of Cooperative Learning Groups***

Traditionally, mathematics has been taught as a “solo,” isolated activity, yet in business and industry mathematicians often work in teams to solve problems and attain common objectives (Steen, 1989). Allowing students to work in partner pairs or cooperative groups affords them the opportunity to develop social and communication skills while working with peers.

Cooperative learning environments, characterized by students working together and interacting with each other, contribute to internalizing concepts by forcing the students to defend their views against challenges brought by their peers. The value of this approach is supported by the work of Vygotsky [(1934)(1986)] who discussed the increasingly interrelated nature of language and cognition as children grow.

Cooperative learning groups are heterogeneous, and everyone must work together for the common good of all. Students who understand the concept being discussed are responsible for explaining it to those who do not understand. When using learning pairs or cooperative groups, teachers must consider new ways of evaluating performance to ensure the success of instructional objectives.

### ***The Role of Assessment***

Making changes in the content and methods of mathematics instruction also requires making changes in why and how students’ work is assessed. Evaluation is an integral part of instruction and not limited to grading and testing. There are at least four reasons why teachers collect evaluation information:

- to make decisions about the content and methods of mathematics instruction
- to make decisions about classroom climate
- to help in communicating what is important
- to assign grades.

In other words, assessment includes much more than marking right and wrong answers. It “must be more than testing; it must be a continuous, dynamic, and often informal process” (NCTM 1989, p. 203). The *Curriculum and Evaluation Standards* recommends that teachers use a variety of types of evaluation: (1) *observing and questioning students* (2) *using assessment data reported by students*; (3) *assessing students’ written mathematics work*; and (4) *using multiple-choice or short-answer items*. Use of these multiple methods of collecting assessment data will contribute to a thorough evaluation of students’ work. *Principles and Standards for School Mathematics* (National Council of Teachers of Mathematics, 2000) states: “Assessment should support the learning of important mathematics and furnish useful information to both teachers and students.” NCTM (1995) identified the following six standards to guide classroom assessment:

Standard 1: Assessment should reflect the mathematics that all students need to know and be able to do.

Standard 2: Assessment should enhance mathematics learning.

Standard 3: Assessment should promote equity.

Standard 4: Assessment should be an open process.

Standard 5: Assessment should promote valid inferences about mathematics learning.

Standard 6: Assessment should be a coherent process.

Implementing the assessment process in the *TEKSING TOWARD STAAR* Middle School Lessons may result in significant changes in how teachers, students and parents view and use assessment as a tool toward student understanding and use of mathematics. Teachers will assess frequently to monitor individual performance and guide instruction.

One intent of the *TEKSING TOWARD STAAR* Lessons is to provide middle school teachers with a structure for instruction that incorporates characteristics of a good mathematics learning environment and the role of assessment as a starting point for student understanding and mastery of the TEKS. Another intent is to provide students with a structure for learning that involves understanding and implementation of “math that matters” in the real world today and in their future.

# CURRICULUM COMPONENTS

Following is an overview of each of the critical components of the *TEKSING TOWARD STAAR* lessons.

## STUDENT PROFILE BOOK

Recording and analysis of data is a critical component of the *TEKSING TOWARD STAAR* Lessons. Recording in a Student Profile Book by each individual student should occur on a regular basis. End of lesson Mini-Assessments, Six Weeks Assessments, Benchmark Assessments and/or Spiraled Practices are examples of data that might be recorded by a student.

## SCOPE AND SEQUENCE

Each six weeks curriculum begins with a Scope and Sequence. The Scope and Sequence provides information for teachers, students, and parents regarding the focus of each *TEKSING TOWARD STAAR* lesson. This guide includes a Scope and Sequence for Six Weeks 1 - 5.

## SIX WEEKS ASSESSMENT/REVIEW

Each six weeks has an open-ended review that can be used as part of the classroom instruction or as homework. Each six weeks ends with a multiple-choice assessment designed to assess all TEKS in lessons from the entire six weeks. This assessment will enable the teacher and student to evaluate student progress toward understanding and use of the skills and concepts for the TEKS addressed during the six weeks. Students should record all work and thinking in written format for each question on the assessment. Students should record individual data on the Student Profile Booklet.

## LESSON COMPONENTS

Following is an overview of the various instructional materials contained in the lessons students will experience during the school year.

## BACKGROUND INSTRUCTIONAL ACTIVITY

Each Instructional Activity in a lesson is specific to TEKS or major pieces of a TEKS. The introductory Instructional Activity(ies) in each lesson is provided to students in a whole class environment with written and illustrated visual aides for whole class instruction. The teacher places visual information in a large group viewing format such as an overhead projector or computerized projection device and leads an informational session designed to provide students with mathematics skills and vocabulary necessary for students to complete the Student Activity(ies) and Problem-Solving activity(ies).

Each student records the critical information from the Instructional Activity on their individual Math Notes page(s). Students record as much information as they choose. The information should be recorded in the student's own "words," "symbols," and pictures or diagrams.

Only minor discussion occurs during the Instructional Activity. This portion of the lesson is designed as an information-giving time. Students are asked to hold most questions until the Instructional Activity portion of the lesson is completed and they begin the Student Activity portion so that the teacher can meet needs on a partner-pair or individual basis.

The teacher leaves the Instructional Activity written information in a place where students can view it later if they find the need to take additional notes.

### **STUDENT ACTIVITY**

A Student Activity(ies) follows each Instructional Activity. Students often work in pairs or small groups to complete a Student Activity; however, each student completes his or her own activity page(s). Math Notes are utilized to enable students to successfully complete the activity. If students did not take notes on material they need to complete the activity, the teacher invites them to view the Instructional Activity written information and to take more detailed notes.

Various partner pairs or small groups are assigned portions of the Student Activity for whole-class discussion. Before students begin the activity, the teacher informs the class of the time allotted for completion of the activity. Time is sometimes called even if all partner pairs or small groups have not completed the activity. Whole class discussion then begins with the partner pairs or small groups that were given assignments to lead the discussion. Students who did not complete the activity may complete the activity at this time by recording in a different color pencil or pen.

The Student Activity is **not** designed to be recorded as a grade based on correct answers, but may be recorded as a holistic score. An example of a holistic scoring scale follows:

- 1 = no understanding evident
- 2 = minimal understanding evident
- 3 = mostly understood or slight mathematical errors
- 4 = complete understanding evident and no mathematical errors

A variation of a Student Activity is included in most lessons. The teacher's notes for these activities include teacher questions posed before and during the activity. The teacher actively looks and listens to student work during the activity. The Student Activity designed as an active, involved, hands-on activity for all students and is often completed as a partner-pair or a group of four students.

### **PROBLEM-SOLVING ACTIVITY**

The Problem Solving activity(ies) are the next component of each lesson and contain problem-solving problems. Students may be assigned to work with a partner or in small groups, but each student must complete an individual Open-Ended student page. Students may utilize Math Notes and Student Activity pages while completing the Open-Ended problem.

The teacher sets a time limit prior to students' beginning the Problem-Solving problem. The students will be provided the 10 questions that will be used on all Problem-Solving activities. Partner pairs are assigned specific "share" portions of the activity. The teacher calls time and the partner pairs guide class discussion on their "share" assignments. Students who did not complete the activity prior to the time limit may record on their individual papers during the discussion time but must record in a different color.

The Problem-Solving Activity is designed to be recorded as a portion of a grade. A holistic score may be recorded for each student. An example of a holistic scale follows:

- 1 = no understanding evident
- 2 = minimal understanding evident
- 3 = mostly understood or slight mathematical errors
- 4 = complete understanding evident and no mathematical errors
- 5 = goes beyond and extends understanding

### **SKILLS AND CONCEPTS HOMEWORK**

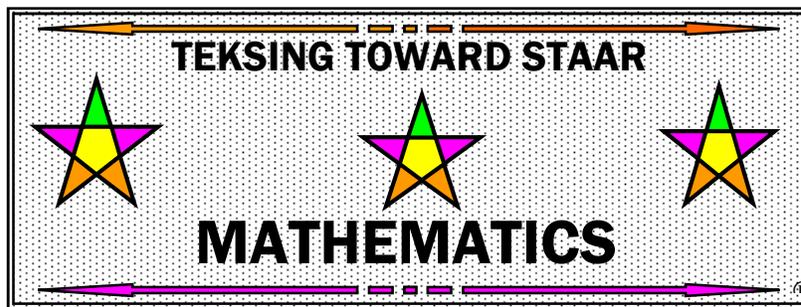
Students will usually be assigned a homework assignment each day of a lesson. If a lesson is more than one instructional day in duration, more than one Homework assignment will be given. Each homework assignment includes 5 open-ended questions. Students should show all work on their homework assignment.

### **MINI-ASSESSMENT**

Each lesson ends with a Mini-Assessment composed of 10 questions. The mini-assessment is completed individually by each student and graded by the teacher. No assistance is given by the teacher during the time allotted for completion of the Mini-Assessment. Most students will be able to complete the Mini-Assessment in approximately 20 minutes.

Students must show all work to answer each problem on the Mini-Assessment. Students should record data from each Mini-Assessment in their Student Profile Book.

As you can see from the overview of the curriculum, *TEKSING TOWARD STAAR* Lessons are extremely comprehensive, are research based and are designed to accelerate mathematics instruction for all students and align classroom instruction with state and national standards. As such, the program should routinely follow the guidelines described above in order to maximize the effectiveness of the curriculum for all students. However, parents should understand that individual teachers may use instructional activities other than those in this program. Teachers may add or delete activities depending on the ebb and flow of their individual classrooms.



## **PARENTAL ROLES, COMMON QUESTIONS AND ANSWERS** **Grade 7 Mathematics Lessons**

### **PARENTAL ROLES**

As a parent, you are extremely interested in your child's education. When parents of middle school and junior high mathematics students work to help their children, they often discover a feeling that "this is not the math I encountered as a seventh grader" and begin to ask themselves what they can do to help their child. Often, parents find it difficult to decide what is "best" when helping their children. Some of the questions parents ask often include:

- *How much help should I give my child?*
- *What if I don't remember (or recognize) some of the math I learned in school?*
- *How can I help my child prepare for tests and other assessments?*
- *How can I help my child discover that math can be fun and doesn't need to be scary?*
- *How can I communicate with my child's teacher to find out what my child should learn?*
- *How can I communicate with my child's teacher to find out how my child is progressing with the understanding of math?*

A successful parent often takes on many roles in the process of parenting. The following roles are involved in helping your child become the best mathematics learner possible.

#### **Role 1: Tutor**

As a tutor, a parent can help with the practice and memorization that are part of getting a firm foundation for truly understanding many math concepts. In middle school mathematics students should practice conversion from decimals to fractions to percents, and then they should finally memorize a specific set of conversions that will enable them to begin to focus on problem solving rather than the skill of conversion. Your child's teacher will give him/her a specific set of conversions that should be memorized and a time line for the process of memorization.

You can also help your child learn about math skills and concepts he or she may have trouble understanding and applying. This guide provides background information to help your child with each lesson. You should start by helping your child work through the information and examples as they are presented in the background information, but you might think of another way to help your child understand that works even better.

## **Role 2: Role Model**

Even if you had a difficult time with math or did not like math when you went to school, try to keep a very positive attitude about the math your child is learning this year. Sometimes it is easy to give the impression that it is OK for your child to do poorly in math when you talk about your own experiences with learning. Instead, focus on how often you use math in your everyday life – discuss situations like comparing prices in a store, balancing your checkbook, setting up newly-purchased electronic equipment, or estimating the cost of paint or flooring for a room in your home.

Share examples of times when you need to stop and think about a problem before solving it. Ask your child about the Problem-Solving Plan he or she is using in the math curriculum (this is explained in the very first lesson of the year). Talk to your child about the fact that some of your real-world problems are harder to solve than others, and that you end up spending more time on those problems and checking your work several times in several different ways. Discuss with your child how solving a very difficult problem is very satisfying, even though it takes a lot of time and hard work.

## **Role 3: Learning Facilitator**

Your child may be very independent and be able to be very successful in math without your help at home. However, be sure to question your child daily about the lesson and homework, and make sure your child begins to review for the Six Weeks Assessment by the end of the fifth week of each six weeks. Also, keep reminding your child that you are always ready to help when needed, or you will find someone else who will help.

## **Role 4: Teaching Partner**

By the time your child reaches middle school or junior high, he or she will probably have a different teacher for each subject on his or her class schedule. The math teacher does not have much time with each class each day and often has many more students than elementary teachers. Many math teachers teach 125 or more students each year, so it may take several weeks or even months for the math teacher to really get to know each individual student. There may be things you have learned about your child's approach to learning that would be helpful for the teacher to know. For example, your child may learn better by "doing an activity" than "taking notes or reading" about math. It is very important to provide your child's teacher with as much information as you can.

Knowing what is being taught and what your child is expected to learn is also very important as a parent. Your child's teacher will probably share information about the curriculum at an Open House early in the school year. If you are not able to attend the Open House, make sure that you communicate with the teacher that you would like information about the curriculum for the year so that you can reinforce the curriculum at home. Let the teacher know that you both have the same goal – to help your child learn to understand and successfully use math.

### **Role 5: Home Learning Environment Creator**

Work with your child to find a place at home with good lighting and near enough to you or someone else to answer questions, help your child stay focused, and provide help if needed. Find a location with no distractions (if there are distractions in the room, your child may choose to work with a soothing music CD and earphones). Make sure the location has room to spread out all the tools and supplies (paper, pencils, pencil sharpener, erasers, graph paper, compass, protractor, scissors, centimeter ruler, and inch ruler, calculator).

Provide encouragement for your child to utilize the space on an almost daily basis. Make homework a part of your child's daily routine - after at least a 30-minute break from the school day - and long before late evening. Help your child get started and stay focused if necessary. Encourage and allow your child to take a five-minute break every 20 minutes while completing homework.

### **Role 6: Homework Helper**

Homework is an extremely useful parental tool for assessing a child's progress in math. Homework provides opportunities for a parent to observe a child's comfort level and understanding of math skills and concepts. Following are steps a parent can take to help their child learn the math curriculum during the school year:

- Step 1: Begin by reading the background information in this guide for each lesson.
- Step 2: Ask your child to review the Math Notes taken during class for this lesson.
- Step 3: Review any missing or incomplete background information with your child.
- Step 4: Ask your child to describe each of the 5 homework problems to you in his or her own words.
- Step 5: Ask your child to describe a process that can be used to answer each problem.
- Step 6: After your child has solved the homework problems, ask if there is another way each problem might be solved. Share a different way you may have thought of, but remember that the way you learned to solve similar problems may or may not help your child understand the problem. Try not to value one method that works more than another method that also works. In mathematics, there are often several good ways to solve the same problem.
- Step 7: Review your child's work. Praise your child for correct answers, then ask your child to redo any of the problems that were incorrect. Ask your child to explain his/her work as each problem is reworked. If the same errors are made again, your child probably does not understand the concept and should go back to his/her Math Notes for a review.
- Step 8: If your child is having difficulty understanding homework, make sure he/she makes time in the daily schedule to attend tutorials offered by the teacher or the school.
- Step 9: Review the previous day's homework with your child and/or review your child's Mini-Assessment after the teacher has graded it and returned it to your child.

Step10: Immediately contact your child’s teacher and request a phone or in-person conference if your child appears to have difficulty for more than 3 days, or does not bring home a homework assignment for more than 2 days, or does not share graded Mini-Assessments with you on a regular basis.

You may have questions and we will try to help you with some answers to common questions on the next several pages of this guide.

## COMMON QUESTIONS AND ANSWERS

The following questions from middle school and junior high parents are very common. Following each question is a brief answer.

**Question 1: Why should my child be using a calculator at school and at home? Isn’t he or she supposed to be learning how to do calculations?**

**ANSWER:** The grades 6-8 TEKS state the following:

**“Problem solving in meaningful contexts, language and communication, connections within and outside mathematics, and formal and informal reasoning underlie all content areas in mathematics. Throughout mathematics in Grades 6-8, students use these processes together with graphing technology and other mathematical tools such as manipulative materials to develop conceptual understanding and solve problems as they do mathematics.”**

Your child should be using a calculator as a tool in grades 6-8 in Texas. A good rule is to use calculators at home as they are used at school. Sometimes the main purpose of your child’s math lesson and homework is to practice computations. For example, your child may be learning how to divide decimal numbers in a lesson at school, so the homework assignment should be done without a calculator. However, your child could check the homework using a calculator, then go back and redo any incorrect answers.

The TEKS, however, are rarely based on calculations. Many times the main purpose of a lesson is to practice solving non-routine problems. For example, if the computation is messy and the focus of the lesson and homework is to graph data, then your child should use the calculator. However, please make sure your child does have the skills necessary to do the computation by hand if he or she is not allowed to use a calculator.

If you are ever in doubt about when to allow your child to use a calculator at home for homework, please contact your child’s teacher.

**Question 2: My child has not been given a textbook, or says he or she doesn't need to use a textbook to do homework. I'd like to help him or her review from time to time, or help him study for tests, but I am not even sure what topics or TEKS are being presented in class or have been presented in class.**

**ANSWER:** Refer to the Scope and Sequence in this guide. Your child should be able to help you identify current and past TEKS and topics presented during class. Look at the top of each homework page or curriculum page your student brings home. The TEKS focus for the lesson is always listed at the top of each page.

Make sure your child is keeping Math Notes, Student Activities, Problem-Solving problems, returned homework assignments, and returned Mini-Assessments in a notebook in an organized manner. You should be able to ask your child for his math notebook at any time and review any of the material with your child. Remember to review the math background in this guide if you need to. *If you are really trying to play the role of tutor for your child, you should both be able to refer to his or her work in order to choose areas of weakness for a more focused review.*

**Question 3: Often my child rushes through the math homework and makes many careless errors, then asks me to check the homework instead of checking it himself. How can I make my child more responsible for the work?**

**ANSWER:** Try to convince your child not to rush through the homework. There are only 5 problems so that students will have time to really think about the questions and do a good job completing the assignment with very few errors. Help your child understand that the teacher is giving fewer homework problems, therefore the teacher expects to see all the student's work to answer each problem, and also evidence the student has checked all answers to make sure they are accurate.

Offer to look over the homework and tell your child which problems contain errors. Your child should then check to find the incorrect answers. Eventually, your child should begin to slow down and be more careful when realizing that finding and correcting careless mistakes takes a lot more time than doing careful work in the first place.

**Question 4: My child asks for help with homework, but what is really being asked is for me to do the work. How much help should I give?**

**ANSWER:** Decide whether there is some non-math reason for your child's request for help. Your child could actually be overtired or would rather be doing something other than homework – if either of these is the case, try changing the routine homework time.

If your child really doesn't understand how to do the problem at all, first take a blank piece of paper and do the problem by yourself with your child being able to see your work as you do it (remember to refer to the background information for the lesson in this guide if you need help). Show every step and explain to your child what you are doing as you record your work. Next, remove the paper and ask your child to redo the same problem on the actual homework sheet, explaining each step to you just as you did for your child earlier.

If your child is still having difficulty, try recording the problem and your solution on another sheet of paper, this time leaving out parts of the solution. Have your child fill in the missing information.

One of the hardest jobs we have as a parent is to be extremely patient and take the time to work with our children, not take the easy, faster way out and do the work for our children.

If your child still doesn't seem to understand, work with your child to write a note to the teacher explaining the problem and promising to complete the homework assignment as soon as the teacher has time to provide additional help such as tutorials during, before and/or after school. Include all the work that you and your child did to try to solve the problem.

**Question 5: My child is very independent and doesn't want me to be involved with math homework. However, sometimes the grade given on the assignment or assessment shows that my child didn't really understand a lesson. What can I do?**

**ANSWER:** A major goal of all parents is to have a child grow into an independent adult. Don't discourage independence. A good goal is to have your child completely independent during homework time by the beginning of grade 9.

When your child finishes the homework, ask if you can check it over and ask your child to explain how one or two of the problems were solved. The explanation can help you decide if your child understands the main concepts. If your child does not want your help looking over the homework to find careless errors, then leave the finding of homework errors to the teacher. Your main concern is that your child understands the main concepts – and if you decide your child does not, then send

them back to the Math Notes taken in class and review the material in this guide in the background information for the lesson.

**Question 6: What should I do if my child brings home an overwhelming amount of homework or no homework at all?**

**ANSWER:** Ask your child if the teacher actually assigned all the homework to be completed in one night. Often, students forget to do their homework for several days and let it pile up – and often their grade will be penalized as “late work” if they do not complete the homework assignments within a certain number of days.

In general, students should have a math homework assignment each day – or should be studying for the end of six weeks assessment. Communicate with your child’s teacher if there appears to be a lack of homework assignments, or your child is consistently telling you that the homework was done in class, or your child comes home with an overwhelming amount of homework – remember – each homework assignment includes only 5 problems to complete.

**OTHER QUESTIONS???? – please contact your child’s math teacher – if the teacher can’t answer your question, feel free to contact the curriculum author at the following e-mail address:**

Juanita Thompson      JThom3250@sbcglobal.net

# **Student Activity**

# Student Activity 1

**Work with your partner to answer the following questions.**

1. Complete the following statements by filling in the blank with an appropriate word or words.

A group of objects or numbers is called a \_\_\_\_\_.

A part of a set is called a \_\_\_\_\_.

The set  $\{1, 2, 3, 4, 5, 6, \dots\}$  is called the set of \_\_\_\_\_.

The set  $\{\dots-6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, \dots\}$  is called the set of \_\_\_\_\_.

The set of numbers that can be expressed as the ratio of two integers is the set of \_\_\_\_\_ numbers.

A terminating decimal is a decimal that \_\_\_\_\_.

A repeating decimal is a decimal that \_\_\_\_\_.

$0.\bar{9}$  is a \_\_\_\_\_ decimal and 1.4 is a \_\_\_\_\_ decimal.

2. Place a  $\checkmark$  in each column that names a set the given number belongs to.

	Rational Number	Integer	Whole Number	Counting Number
-16				
0				
1.5				
$\frac{21}{4}$				
$-4.\bar{2}$				
$-35\frac{2}{3}$				
1,250				
$0.\bar{12}$				

3. Name 3 integers that are NOT whole numbers.

\_\_\_\_\_

4. Name 3 rational numbers that are NOT integers.

\_\_\_\_\_

5. Name a rational number that would be between 3 and 3.1 on a number line.

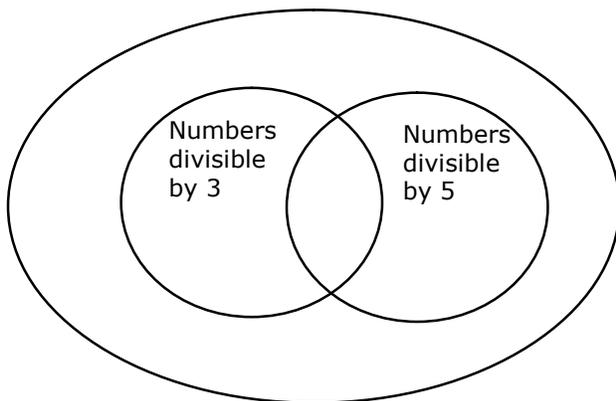
6. Draw a Venn diagram that shows the relationship among rational numbers, integers, whole numbers, and natural numbers.
7. Place the following numbers in the appropriate set on the Venn diagram you drew in Question 6.
- 217      -4      1.1       $\frac{21}{3}$       3      125       $0.\overline{4}$        $-2\frac{1}{2}$
8. Identify each statement below as T(true) or NT(not true).
- \_\_\_\_\_ 1. All prime numbers are integers.
- \_\_\_\_\_ 2. All decimals are rational numbers.
- \_\_\_\_\_ 3. All whole numbers are counting numbers.
- \_\_\_\_\_ 4. All whole numbers are integers.
9. Name 2 counting numbers that will be between 3 and 6.5 on a number line.
10. Using a W for whole numbers, I for integers, and R for rational numbers, identify all the sets of numbers that have members in the given set.
- $\{-1, -3, -14, -13\}$  \_\_\_\_\_
- $\left\{\frac{22}{7}, 3.\overline{14}, 4, 0\right\}$  \_\_\_\_\_
- $\left\{-20, -1.1, \frac{4}{3}, -3\right\}$  \_\_\_\_\_

## Student Activity 2

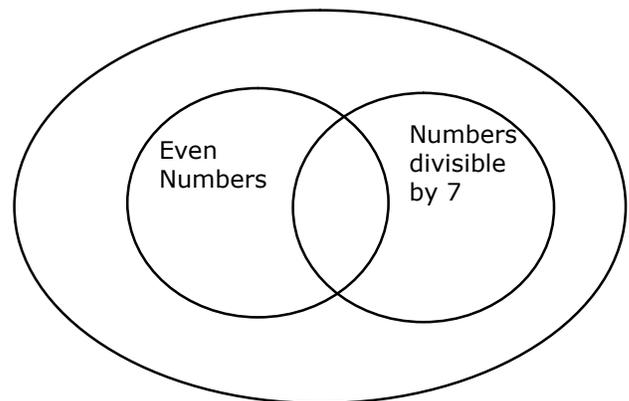
Work with your partner to answer the following questions.

1. Identify the set of numbers that best describes each situation
  - The amount of the ingredients used to make brownies
  - The number of homeruns hit by a baseball team during the last game of the season
  - A board game has a spinner with 3 sections- Lose your Turn, Move Forward, Move Backward and a number cube with the numbers 1-6. The number of moves you make after a spin and a roll
  - The number of students on a school bus when it arrives at school in the morning
  - The number of seconds recorded for the times of the participants running the 100 meter dash at a track meet
  - The balance in a person's check register
  - The height of a person in centimeters
2. How can you show the relationship among the set of rational numbers, integers, whole numbers, and counting numbers?
3. Fill in the following Venn diagrams with the counting numbers 1 to 20.

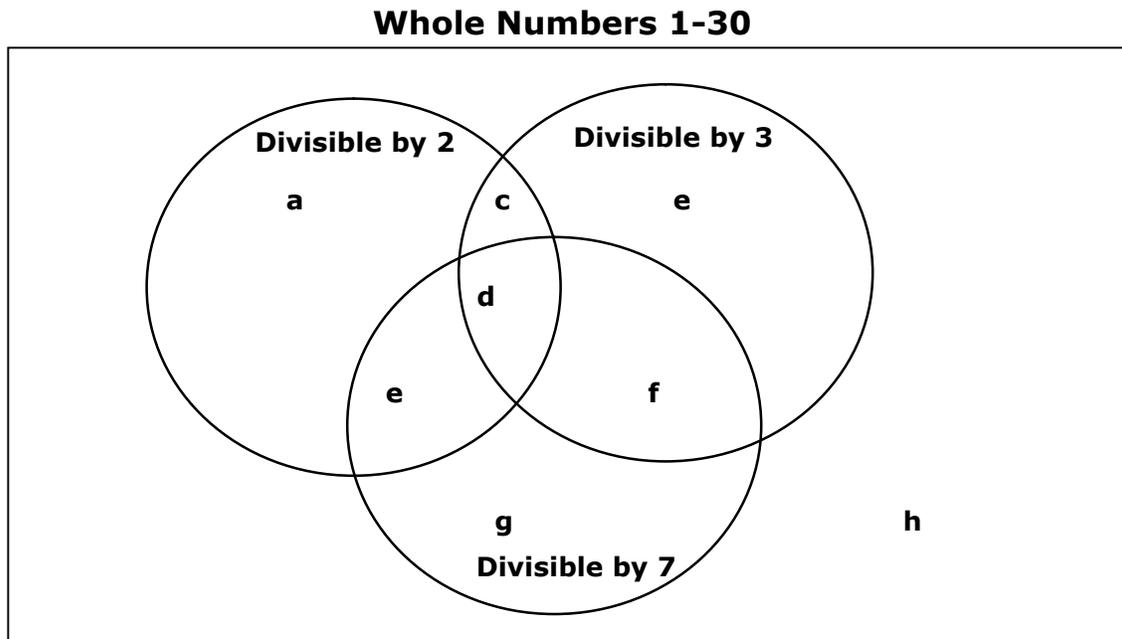
Counting Numbers 1 to 20



Counting Numbers 1 to 20



4. Look at the Venn diagram below. It contains the set of whole numbers 1-30.



- Describe verbally the numbers that would be in the section labeled a.

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- List the number(s) that would be in section a.
- Describe verbally the numbers that would be in the section labeled c.

---

- List the number(s) that would be in section c.
- Describe verbally the numbers that would be in the section labeled g.

---

- List the number(s) that would be in section g.

- Describe verbally the numbers that would be in the section labeled d.

---

- List the number(s) that would be in section d.
- Describe verbally the numbers that would be in the section labeled b.

---

- List the number(s) that would be in section b.
- Describe verbally the numbers that would be in the section labeled h.

---

- List the number(s) that would be in section h.
- Describe verbally the numbers that would be in the section labeled f.

---

- List the number(s) that would be in section f.
- Describe verbally the numbers that would be in the section labeled e.

---

- List the number(s) that would be in section e.

Are all the whole numbers 1-30 found in at least one of the sections? Explain your answer.

Were any of the sections empty? Explain

# **Problem-Solving Problem**

# Problem-Solving Questions

**Directions:**

- **Work with a partner.**
- **Write your answers on notebook paper.**
- **Answer questions 1-3.**
- **Complete the solution to the problem(s).**
- **Answer questions 4-10.**

1. What is the main idea of this problem?
2. What are the supporting details in this problem?
3. What skills, concepts, and understanding of math vocabulary are needed to be able to answer this problem?
4. Did this problem involve mathematics arising in everyday life, society, or the work place?
5. What is a good problem solving strategy for this problem?
6. Can you explain how you used any math tools, mental math, estimation, or number sense to solve this problem?
7. Did this problem involve using multiple representations (symbols, diagrams, graphs, math language)?
8. Did you use any relationships to solve this problem?
9. How can you justify your solution to the problem?
10. How can you check for reasonableness of your solution to this problem?

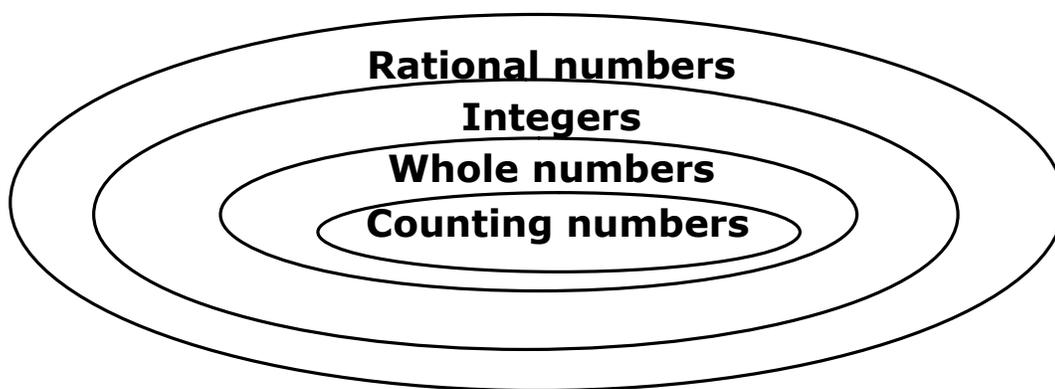
**Problem-Solving 1**

**Problem 1:** Which of the following statements are true? Use T or NT.

- \_\_\_\_\_ 1. All integers are whole numbers
- \_\_\_\_\_ 2. Any rational number can be expressed as the ratio of two integers.
- \_\_\_\_\_ 3. Some decimal numbers are not rational numbers.
- \_\_\_\_\_ 4. All integers are also rational numbers.
- \_\_\_\_\_ 5. The set  $\{8, 8.5, 10, -23\}$  are all rational numbers.
- \_\_\_\_\_ 6. The set  $\{-3, 19, 20, 0, -1\}$  are all integers.

For any statement you listed as NT, explain your reasoning.

**Problem 2:** Place  $-6$ ,  $0$ ,  $3.5$ ,  $\frac{12}{4}$ ,  $-3\frac{1}{2}$  and  $0.\overline{45}$  in the appropriate place on the Venn diagram.



## Problem-Solving 2

**Problem 1:** Identify the set of numbers that best describe the situations below.

- Numbers used in a phone number

\_\_\_\_\_

- Golf scores on a leaderboard

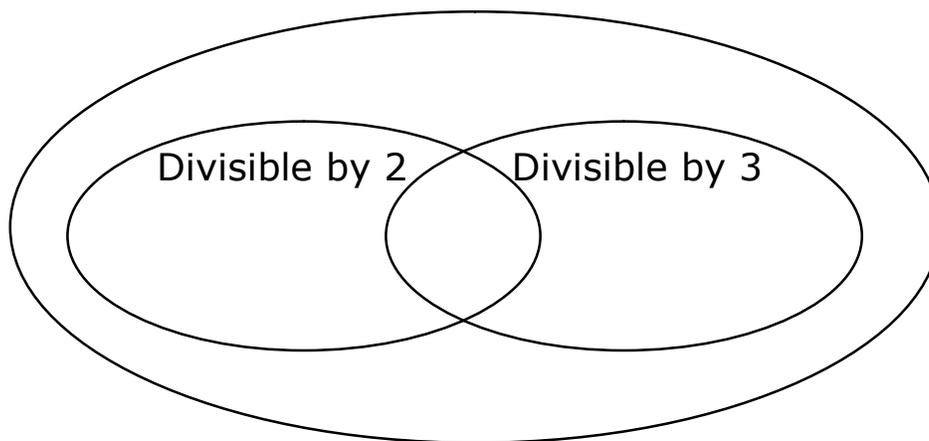
\_\_\_\_\_

- Total cost of grocery store purchases

\_\_\_\_\_

**Problem 2:** Place the counting numbers 1-18 on the Venn diagram below.

Counting Numbers 1-18



# Homework

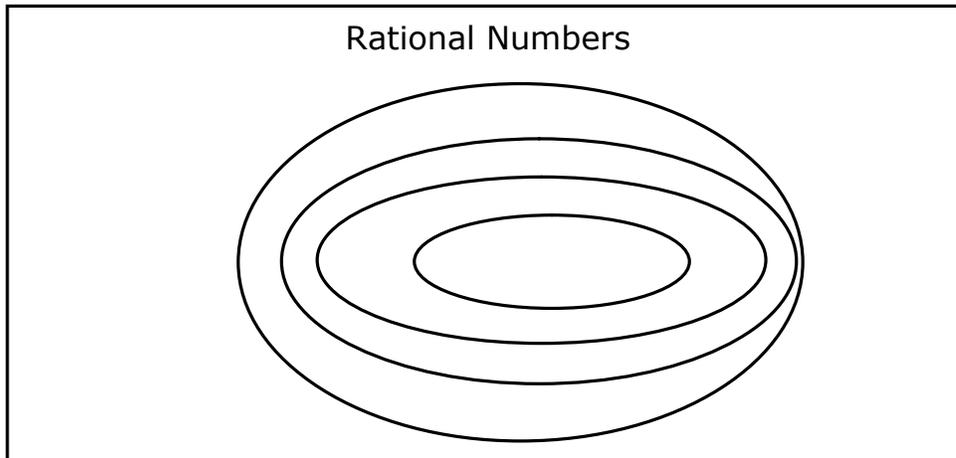
NAME \_\_\_\_\_

DATE \_\_\_\_\_

SCORE \_\_\_/5

## 7.2A Skills and Concepts Homework 1

1. Fill in the Venn diagram below showing the relationship of rational numbers, integers, whole numbers, and counting numbers



2. Place a  $\checkmark$  in each column that the given number belongs to.

	Rational Number	Integer	Whole Number	Counting Number
-22				
-3.1				
113				
$\frac{5}{8}$				
0.242424...				

3. Name a whole number that is NOT a counting number. \_\_\_\_\_

4. Name 3 rational numbers that are NOT positive.

\_\_\_\_\_

5. Name a rational number that is located between 31.5 and 31.6 on a number line. How do you know it is rational?

NAME \_\_\_\_\_

DATE \_\_\_\_\_

SCORE \_\_\_/5

## 7.2A Skills and Concepts Homework 2

1. Identify the set of numbers that best describes each situation.

- The number of miles you could walk in 30 minutes
- Possible number of cookies in a cookie jar
- Number of fish caught in an hour of fishing
- Scores of the top 5 golfers on a leaderboard

2. Explain how the set of integers differs from the set of counting numbers.

3. What is a composite number?

Are composite numbers counting numbers? Explain

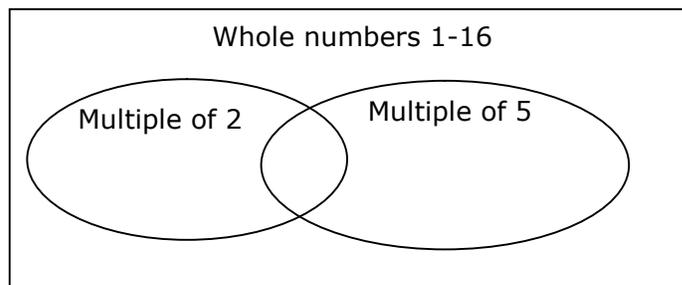
4. Identify which set of numbers are listed below.

$\{\dots -3, -2, -1, 0, 1, 2, 3, \dots\}$  \_\_\_\_\_

$\{0, 1, 2, 3, \dots\}$  \_\_\_\_\_

$\{0, 2, 4, 6, \dots\}$  \_\_\_\_\_

5. Fill in the Venn diagram below with whole numbers 1-16.



# Mini-Assessment

NAME \_\_\_\_\_

DATE \_\_\_\_\_

SCORE \_\_\_\_/10

## Mini-Assessment 7.2A

1. Which number does NOT represent an integer?

- A 3
  - B 20.1
  - C -10
  - D  $\frac{20}{4}$
- 

2. Which describes a rational number?

- F Any number found on a number line
  - G All numbers greater than 0
  - H Any number that can be expressed as the ratio of two integers where the denominator is not 0
  - J Any decimal number
- 

3. Which statement is true?

- A Every rational number is an integer.
  - B Every whole number is a counting number.
  - C Every integer is a whole number.
  - D Every whole number is a rational number.
- 

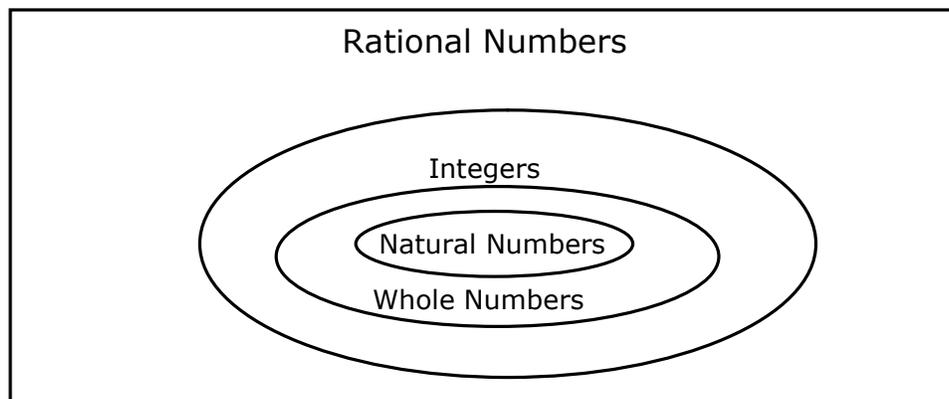
4. A coin collection contains nickels, dimes, and quarters. Which set of numbers would be used to describe the number of dimes in the collection?

- F Positive rational numbers
- G Counting numbers
- H Integers
- J Rational numbers

5. Which of the following does NOT represent a set of integers?

- A**  $\{3, 4, 6, 5\}$
- B**  $\left\{\frac{4}{3}, 12, 3.0, 9\right\}$
- C**  $\left\{\frac{12}{4}, \frac{-8}{4}, \frac{9}{3}, 15\right\}$
- D**  $\{0, 13, 65, 100\}$
- 

6. This Venn diagram shows the relationship of the subsets of the real number system.



Which of the following sets would belong to the natural numbers?

- F**  $\{6, -5, 1.25\}$
- G**  $\{2, 4, 0.\bar{3}\}$
- H**  $\{-8, 4, 13, 25\}$
- J**  $\left\{\frac{16}{4}, 8, 7, 9\right\}$
- 

7. Which statement is NOT true?

- A**  $-15$  is a whole number and an integer.
- B**  $-15$  is an integer and a rational number.
- C**  $-15$  is a rational number but is not a whole number.
- D**  $-15$  is not a whole number.

8. Which number is a rational number that is NOT a whole number?

**F** 2

**G** 12

**H**  $\frac{30}{10}$

**J**  $\frac{17}{5}$

9. Rational numbers are a dense set. This means that between any two rational numbers on a number line there is another rational number. Which rational number is between 2.23 and 2.24 on a number line?

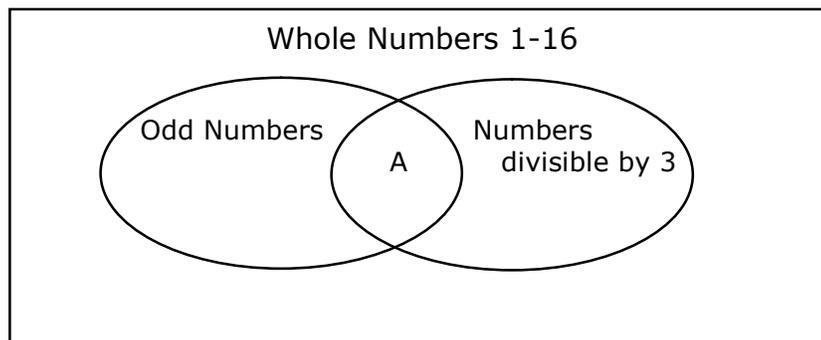
**A** 2.241

**B** 2.236

**C** 2.256

**D** 2.223

10. The Venn diagram below contains the whole numbers 1-16.



Which of the following lists names the numbers that would be located in section A?

**F** 1, 3, 6, 9, 12, 15

**G** 3, 6, 9, 12, 15

**H** 3, 9, 12

**J** 3, 9, 15

# **Problem-Solving Plan**

# Problem-Solving Model

Step	Description of Step
1	<p><b>Analyze the given information.</b></p> <ul style="list-style-type: none"> <li>• Summarize the problem in your own words.</li> <li>• Describe the main idea of the problem.</li> <li>• Identify information needed to solve the problem.</li> </ul>
2	<p><b>Formulate a plan or strategy.</b></p> <ul style="list-style-type: none"> <li>• Draw a picture or diagram.</li> <li>• Guess and check.</li> <li>• Find a pattern.</li> <li>• Act it out.</li> <li>• Create or use a chart or table.</li> <li>• Work a simpler problem.</li> <li>• Work backwards.</li> <li>• Make an organized list.</li> <li>• Use logical reasoning.</li> <li>• Brainstorm.</li> <li>• Write a number sentence or an equation</li> </ul>
3	<p><b>Determine a solution.</b></p> <ul style="list-style-type: none"> <li>• Estimate the solution to the problem.</li> <li>• Solve the problem.</li> </ul>
4	<p><b>Justify the solution.</b></p> <ul style="list-style-type: none"> <li>• Explain why your solution solves the problem.</li> </ul>
5	<p><b>Evaluate the process and the reasonableness of your solution.</b></p> <ul style="list-style-type: none"> <li>• Make sure the solution matches the problem.</li> <li>• Solve the problem in a different way.</li> </ul>

**TEKSING TOWARD STAAR SCOPE AND SEQUENCE**  
**Grade 7 Mathematics**  
**Six Weeks 1**

Lesson	TEKS-BASED LESSON
<b>Lesson 1</b>	<b>7.2A</b> /extend previous knowledge of sets and subsets using a visual representation to describe relationships between sets of rational numbers.
<b>Lesson 2</b>	<b>7.3A</b> /add, subtract multiply and divide rational numbers fluently
<b>Lesson 3</b>	<b>7.3B</b> /apply and extend previous understandings of operations to solve problems using addition, subtraction, multiplication, and division of rational numbers
<b>Lesson 4</b>	<b>7.5A</b> /generalize the critical attributes of similarity, including ratios within and between similar shapes  <b>7.5C</b> /solve mathematical and real-world problems involving similar shapes and scale drawings
<b>Lesson 5</b>	<b>7.6A</b> /represent sample spaces for simple and compound events with and without technology  <b>7.6E</b> /find the probabilities of a simple even and its complement and describe the relationship between the two.
<b>Lesson 6</b>	<b>7.10A</b> /write one-variable, two-step equations and inequalities to represent constraints or conditions within problems  <b>7.11A</b> /model and solve one-variable, two-step equations and inequalities  <b>7.11B</b> /determine if the given value(s) make(s) one-variable, two-step equations and inequalities true
<b>Lesson 7</b>	<b>7.12A</b> /compare two groups of numeric data using comparative dot plots....by comparing their shapes, centers and spreads
<b>Lesson 8</b>	<b>7.13A</b> /calculate the sales tax for a given purchase and calculate income tax for earned wages
<b>Lesson 9</b>	<b>7.13B</b> /identify the components of a personal budget, including income, planned savings for college, retirement, and emergencies; taxes; fixed and variable expenses, and calculate what percentage each category comprises of the total budget

**Six Weeks 1**  
**Lesson 1**

## Parent Guide

### Six Weeks 1 Lesson 1

For this lesson, students should be able to demonstrate an understanding of how to represent probabilities and numbers. Students are expected to apply mathematical process standards to represent and use rational numbers in a variety of forms.

Students are also expected to extend previous knowledge of sets and subsets using a visual representation to describe relationships between sets of rational numbers.

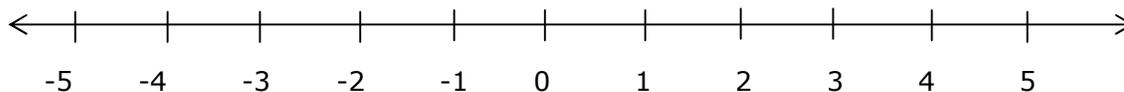
The process standards incorporated in this lesson include:

- 7.1B** Use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution
- 7.1D** Communicate mathematical ideas, reasoning and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate
- 7.1F** Analyze mathematical relationships to connect and communicate mathematical ideas

#### Math Background-Understanding Rational Numbers

A group of items or numbers is called a set. A part of that set is called a **subset**. The set of numbers we use in our every day lives is the set of real numbers. These are the numbers that are located on a number line. One subset of the real numbers is the set of whole numbers. **Whole numbers** are the numbers 0, 1, 2, 3, 4... Each of these numbers has an opposite 0, -1, -2, -3, -4... When the whole numbers and their opposites are joined together the set of **integers** is created.

The set of integers are indicated in set notation as  $\{\dots-4, -3, -2, -1, 0, 1, 2, 3, 4\dots\}$ . These numbers are used to label a number line with the negative numbers located to the left of zero and the positive numbers located to the right of zero.



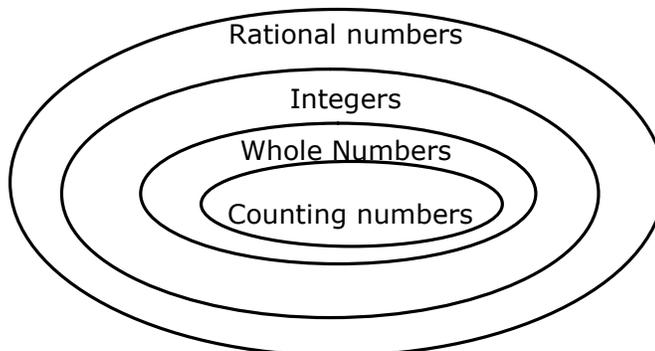
If zero is removed from the set of whole numbers, the set of **natural numbers** or **counting numbers** is created. The natural numbers can be indicated in set notation as  $\{1, 2, 3, 4, 5, 6, 7, \dots\}$ .

The whole numbers, counting numbers, and integers are all subsets of a larger set called the rational numbers. When a number of the form  $\frac{a}{b}$  is created where  $a$  and  $b$  are both integers but  $b \neq 0$ , then

the set of **rational numbers** is created. For example, the ratio of 2 to 3 creates  $\frac{2}{3}$ , so  $\frac{2}{3}$  is a rational number. The ratio of 10 to 2 creates  $\frac{10}{2}$  or 5 which is a whole number as well as a rational number.

A mixed number like  $3\frac{1}{2}$  is a rational number because it can be rewritten as an improper fraction,  $\frac{7}{2}$ , which is the ratio of two integers.

The relationship between these sets of subsets of the real numbers can be shown with a Venn diagram.



This diagram shows that all whole numbers are integers, and all integers are rational numbers. When a set is included completely in another set on the diagram, then all members of the smaller set are also members of the larger set.

Some decimals are rational numbers and some decimals are not rational numbers. If the decimal terminates (ends) OR it has repeating digits, then it is a rational number.

1.25, 1.3, 1.001 are terminating decimals and thus are rational numbers. They can be expressed as a ratio of two integers.

$$1.25 = \frac{125}{100} \quad 1.3 = \frac{13}{10} \quad 1.001 = \frac{1001}{1000}$$

$0.\bar{3}$  and  $0.\bar{6}$  are two of the most commonly used repeating decimals.

$$0.\bar{3} = \frac{1}{3} \quad 0.\bar{6} = \frac{2}{3}$$

2.1357911... and 0.646446444... are examples of decimals that are NOT rational numbers. They do not terminate nor do they have repeating digits.

**Example:** Classify each number by naming the set(s) that it belongs to.

$\frac{1}{4}$  Rational number It is ratio of integers 1 and 4.

-0.15 Rational number It is the ratio of integers -15 and 100

43 Counting number, whole number, integer, and rational number

-13 Integer, rational number

$0.\bar{7}$  Rational number It is the ratio of integers 7 and 9

**Identifying Number Sets Used in Real-World Situations**

Numbers used in real-world situations can be whole numbers, integers, and rational numbers. When identifying the set of numbers that could be used in a particular situation, select the one that gives the most precise set. For example, if counting numbers is the set used to describe a situation, you could say whole numbers, integers, and rational numbers. By using the most specific set, counting numbers, it is understood that the other sets would work also.

**Example:** The number of dimes in a person's pocket

Whole numbers. This set best describes the number of dimes because the person may have 0, 1, 2, 3, ... dimes in his pocket.

**Example:** The lengths of ribbon on 5 spools of ribbon

Positive rational numbers. This set best describes the lengths of ribbon on the spools because the lengths can be numbers like 3.5,  $\frac{1}{2}$ , 3, etc. Measurements must be positive numbers.

There are subsets of various sets of numbers that are described by a characteristic. For example, the whole numbers divisible by 6, are a subset of the whole numbers. This set would be 6, 12, 18, 24, 30, etc. This set could also be called the multiples of 6.

Some other subsets are even numbers, odd numbers, composite numbers, prime numbers, etc. We have learned about these subsets in prior grades.

**TEKSING TOWARD STAAR SCOPE AND SEQUENCE**  
**Grade 7 Mathematics**

**SIX WEEKS 3**

Lesson	TEKS-BASED LESSON	STAAR Category Standard	Spiraled Practice	Student Activity	Problem Solving	Skills and Concepts Homework
<b>Lesson 1</b> ____ days	<b>7.4D/</b> solve problems involving....percents of decrease and percent of decrease and financial literacy problems	Category 2 Readiness	SP 41 SP 42	SA 1 SA 2	PS 1 PS 2	Homework 1 Homework 2
<b>Lesson 2</b> ____ days	<b>7.7A/</b> represent linear relationships using verbal descriptions, tables, ....,that simplify to the form $y = mx + b$ .	Category 2 Readiness	SP 43 SP 44	SA 1 SA 2	PS 1 PS 2	Homework 1 Homework 2
<b>Lesson 3</b> ____ days	<b>7.8C/</b> use models to determine the approximate formulas for the circumference and area of a circle and connect the models to the actual formulas  <b>7.5B/</b> describe $\pi$ as the ratio of the circumference of a circle and its diameter  <b>7.9B/</b> determine the circumference and area of circles	Not Tested  Category 3 Supporting Category 3 Readiness	SP 45 SP 46	SA 1 SA 2	PS 1 PS 2	Homework 1 Homework 2
<b>Lesson 4</b> ____ days	<b>7.6I/</b> determine experimental and theoretical probabilities related to simple and compound events using data and sample spaces	Category 1 Readiness	SP 47 SP 48	SA 1 SA 2	PS 1 PS 2	Homework 1 Homework 2
<b>Lesson 5</b> ____ days	<b>7.8B/</b> explain verbally and symbolically the relationship between the volume of a triangular prism and a triangular pyramid both have congruent bases and heights and connect that relationship to the formulas  <b>7.9A/</b> solve problems involving the volume of ....triangular prisms and triangular pyramids	Not Tested  Category 3 Readiness	SP 49 SP 50	SA 1 SA 2	PS 1 PS 2	Homework 1 Homework 2
<b>Lesson 6</b> ____ days	<b>7.9C/</b> determine the area of composite figures containing combinations of rectangles, squares, parallelograms, trapezoids, triangles, semicircles, and quarter circles	Category 3 Readiness	SP 51 SP 52	SA 1 SA 2	PS 1 PS 2	Homework 1 Homework 2
<b>Lesson 7</b> ____ days	<b>7.9D/</b> solve problems involving the lateral and total surface area of a rectangular prism,...rectangular pyramid,..by determining the area of the shape's net	Category 3 Supporting	SP 53 SP 54	SA 1 SA 2	PS 1 PS 2	Homework 1 Homework 2

**TEKSING TOWARD STAAR SCOPE AND SEQUENCE**  
**Grade 7 Mathematics**

**SIX WEEKS 3**

Lesson	TEKS-BASED LESSON	STAAR Category Standard	Spiraled Practice	Student Activity	Problem Solving	Skills and Concepts Homework
<b>Lesson 8</b> ____ days	<b>7.11C</b> /write and solve equations using geometry concepts, including the sum of the angles in a triangle and angle relationships	Category 3 Supporting	SP 55 SP 56	SA 1 SA 2	PS 1 PS 2	Homework 1 Homework 2
<b>Lesson 9</b> ____ days	<b>7.6G</b> /solve problems using data represented in...dot plots, including part-to-whole and part-to-part comparisons and equivalents	Category 4 Readiness	SP 57 SP 58	SA 1 SA 2	PS 1 PS 2	Homework 1 Homework 2
<b>Lesson 10</b> ____ days	<b>7.12B</b> /use data from a random sample to make inferences about a population  <b>7.6F</b> /use data from a random sample to make inferences about a population	Category 4 Supporting  Not Tested	SP 59 SP 60	SA 1 SA 2	PS 1 PS 2	Homework 1 Homework 2
<b>Review Assessment</b> 2 days	<b>Six Weeks 3 Open-Ended Review</b> <b>Six Weeks 3 Assessment</b>					

**Teacher Notes:**

**Six Weeks 3**  
**Lesson 4**

## Parent Guide

### Six Weeks 3 Lesson 4

For this lesson, students should be able to demonstrate an understanding of how to represent probabilities and numbers. Students apply mathematical process standards to probability and statistics to describe or solve problems involving proportional relationships.

Students are expected to determine experimental and theoretical probabilities related to simple and compound events using data and sample spaces.

The process standards incorporated in this lesson include:

- 7.1A** apply mathematics to problems arising in everyday life, society, and the workplace
- 7.1B** Use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution
- 7.1D** Communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate.
- 7.1E** Create and use representations to organize, record, and communicate mathematical ideas

#### Math Background-Determining Experimental and Theoretical Probabilities using Data

Recall from an earlier lesson, the probability of an event is the ratio of the number of favorable outcomes to the number of possible outcomes.

$$P(\text{event}) = \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}}$$

There are two types of probabilities. One probability is called the theoretical probability. The theoretical probability ratio is still  $\frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}}$ . For example, if you toss a coin, there are 2 possible outcomes, a tail or a head. The theoretical probability of tossing a head is  $\frac{1}{2}$ . The theoretical probability of tossing a tail is also  $\frac{1}{2}$ .

The other type of probability is called the experimental probability. The experimental probability ratio is  $\frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}}$ . The data used in the ratio is based on an experiment of trials.

For example, if you toss a coin 50 times, you could get 25 heads and 25 tails which would match the theoretical probability. However, it is more likely you would get data such as 27 tails and 23 heads. If this were the data you recorded for tossing a penny 50 times, the experimental probability of tossing a head would NOT be  $\frac{1}{2}$ . It would be  $\frac{\text{number of heads}}{\text{number of tosses}}$  or  $\frac{23}{50}$ , which is less than  $\frac{1}{2}$ .

The more trials you have, the more likely the experimental probability will be closer to the theoretical probability. If you tossed a coin 2000 times, you would expect the number of heads and the number of tails to be close to 1000 each or  $\frac{1}{2}$  of the tosses.

Since a probability is a ratio of two values, it can be expressed as a fraction, decimal, or percent. If the probability is  $\frac{1}{2}$ , then it can be written also as 0.5 or 50%.

**Example:** A bag contains 5 red tiles, 6 blue tiles, and 4 yellow tiles. Benny randomly selects a tile from the bag. What is the theoretical probability he will select a red tile? A yellow tile? A blue tile?

$$P(r) = \frac{\text{number of red tiles}}{\text{number of tiles}} = \frac{5}{15} = \frac{1}{3}$$

The theoretical probability of randomly drawing a red tile is  $\frac{1}{3}$  or  $33\frac{1}{3}\%$ .

$$P(y) = \frac{\text{number of yellow tiles}}{\text{number of tiles}} = \frac{4}{15}$$

The theoretical probability of randomly drawing a yellow tile is  $\frac{4}{15}$  or  $26\frac{2}{3}\%$ .

$$P(b) = \frac{\text{number of blue tiles}}{\text{number of tiles}} = \frac{6}{15} = \frac{2}{5}$$

The theoretical probability of randomly drawing a blue tile is  $\frac{2}{5}$  or 40%.

**Example:** A bag contains 5 red tiles, 6 blue tiles, and 4 yellow tiles. Benny randomly selects a tile from the bag. Benny draws a tile from the bag, records its color, and returns the tile to the bag before drawing another tile. He does this 40 times. The results of his experiment are recorded in the table below.

Color	red	blue	yellow
No. of Draws	12	18	10

Based on Benny's data from his experiment, what is the experimental probability the next tile he draws from the bag will be red?

$$P(r) = \frac{\text{number of red draws}}{\text{number of draws}} = \frac{12}{40} = \frac{3}{10} \text{ or } 30\%$$

Based on Benny's data from his experiment, what is the experimental probability the next tile he draws from the bag will be yellow?

$$P(y) = \frac{\text{number of yellow draws}}{\text{number of draws}} = \frac{10}{40} = \frac{1}{4} \text{ or } 25\%$$

Based on Benny's data from his experiment, what is the experimental probability the next tile he draws from the bag will be blue?

$$P(b) = \frac{\text{number of blue draws}}{\text{number of draws}} = \frac{18}{40} = \frac{9}{20} \text{ or } 45\%$$

**Example:** Using the two examples above, compare the theoretical probability to the experimental probability of drawing each color tile.

**RED:**

The theoretical probability of drawing a red is  $\frac{1}{3}$  or  $33\frac{1}{3}\%$ . The experimental probability of drawing a red is  $\frac{3}{10}$  or 30%. The theoretical probability is slightly larger than the experimental probability.

**YELLOW:**

The theoretical probability of drawing a yellow is  $\frac{4}{15}$  or  $26\frac{2}{3}\%$ . The experimental probability of drawing a yellow is  $\frac{1}{4}$  or 25%. The theoretical probability is slightly larger than the experimental probability.

**BLUE:** The theoretical probability of drawing a blue is  $\frac{2}{5}$  or 40%. The experimental probability of drawing a blue is  $\frac{9}{20}$  or 45%. The experimental probability is slightly larger than the theoretical probability.

The sum of the three theoretical probabilities and the sum of the three experimental probabilities must each be 1 or 100%. Use that as a check to make sure you have not miscalculated.

Simple events are when there is one event. The event can be tossing a coin, rolling a number cube, drawing a card, spinning a spinner, etc. Compound events are when you have more than one event occurring. The events could be tossing a coin and spinning a spinner, tossing a coin and rolling a number cube, drawing a card and spinning a spinner, spinning 2 different spinners, etc.

Compound events can be events that are independent events. Independent events are events that the results of one event do NOT affect the results of the other event. An example of independent events is tossing a coin and spinning a spinner. If the results of one event do affect the results of the second event, then they are dependent events. An example of dependent events is drawing 2 tiles from a bag, one at a time, and NOT replacing the first tile before drawing the second tile. The first draw affects the number of tiles in the bag for the second draw.

Where there are two independent events, the probability of certain events occurring is the product of the probability of each event occurring.  $P(A \text{ and } B) = P(A) \times P(B)$

When there are two dependent events, the probability of certain events occurring is the probability of the first event times the probability of the second event occurring given the occurrence of the first event. This is written  $P(A \text{ and } B) = P(A) \times P(B/A)$

**Example:** A bag contains 5 red marbles and 10 blue marbles. You are to select a marble, record its color, replace the marble in the bag, and then draw a second marble. What is the probability you will draw 2 marbles that are red?

The  $P(r) = \frac{5}{15} = \frac{1}{3}$  for the first draw. The  $P(r) = \frac{5}{15} = \frac{1}{3}$  for the second draw.

The  $P(r \text{ and } r) = \frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$ .

These were independent events.

**Example:** A bag contains 5 red marbles and 10 blue marbles. You are to select a marble, record its color, do NOT replace the marble in the bag, and then draw a second marble. What is the probability you will draw 2 marbles that are red?

The  $P(r) = \frac{5}{15} = \frac{1}{3}$  for the first draw. The  $P(r) = \frac{4}{14} = \frac{2}{7}$  for the second draw. (1 red has been drawn and NOT replaced so there are only 4 red marbles now and there are only 14 marbles)

The  $P(r \text{ and } r) = \frac{1}{3} \times \frac{2}{7} = \frac{2}{21}$ .

These were dependent events.

The probabilities of drawing 2 red marbles are not the same for the two examples. Replacing the marble back in the bag before drawing the second marble makes the events independent. Which situation had the greater probability of occurring?

### Determining Theoretical and Experimental Probabilities using Sample Spaces

A **sample space** of an event is a set of all the possible outcomes of the event. The set can be a list, a tree diagram, or a table.

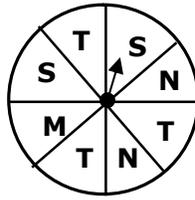
A sample space for tossing a coin is {heads, tails}. A sample space for rolling a 1-6 number cube is the list: 1, 2, 3, 4, 5, and 6.

To determine the probability of an event using a sample space, use the same ratio  $\frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}}$ . Look at the sample space and count the number of favorable outcomes for the numerator of the ratio. Count the total number of entries in the set for the denominator of the ratio.

**Example:** What is the probability of rolling a 6 on a 1-6 number cube?

A sample space for rolling a number cube is {1, 2, 3, 4, 5, 6}. The number of favorable outcomes ( a 6) is 1. The number of entries in the set is 6. The ratio that represents the probability is  $\frac{1}{6}$ .

**Example:** If you spin the spinner below, what is the probability you will spin a T?



A sample space for the spinner is  $\{S, T, S, N, T, N, T, M\}$ . The probability of spinning a T is  $\frac{\text{number of Ts}}{\text{number of outcomes}} \cdot P(T) = \frac{3}{8}$

**Example:** You are rolling a number cube and tossing a coin. What is the probability you will roll a 4 and toss a tails?

A sample space for the number cube is  $\{1, 2, 3, 4, 5, 6\}$

A sample space for tossing a coin is  $\{\text{heads}, \text{tails}\}$

$$P(4) = \frac{1}{6} \quad P(\text{tails}) = \frac{1}{2} \quad P(4 \text{ and tails}) = \frac{1}{6} \times \frac{1}{2} = \frac{1}{12}$$

A sample space for both events could be the list: 1/tails; 1/heads; 2/tails; 2/heads; 3/tails; 3/heads; 4/tails; 4/heads; 5/tails; 5/heads; 6/tails; 6/heads

There are 12 items in the list and 1 in the list is 4/tails. The probability would be  $\frac{1}{12}$ .

**Example:** A dessert store kept a record of the number of slices of apple pie they sold one day last week. They also recorded the choice of topping. The table shows the sample space of the apple pie slices sold that day.

Topping	Whipped Cream	Ice Cream	No Topping
Number Served	32	25	43

What is the probability the next slice of apple pie ordered will have a whipped cream topping?

Total the number of slices served.  $32 + 25 + 43 = 100$ .  $P(\text{WC}) = \frac{32}{100}$  or 32%.