TEKSING TOWARD STAAR

MATHEMATICS®

GRADE 7

Projection Masters

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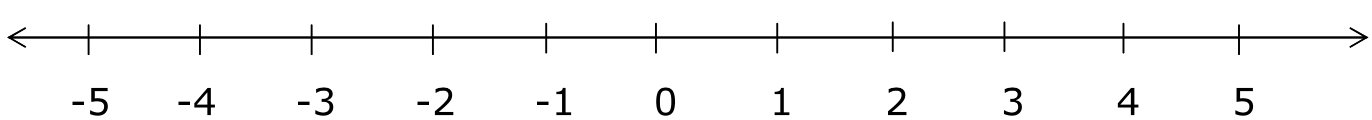
Six Weeks 1
Lesson 1

Understanding Rational Numbers

A group of items or numbers is called a **set**. A part of that set is called a **subset**. The set of numbers we use in our every day lives is the set of **real numbers.** These are the numbers that are located on a number line. One subset of the real numbers is the set of **whole numbers**. Whole numbers are the numbers 0, 1, 2, 3, 4… Each of these numbers has an opposite 0, -1, -2, -3, -4… When the whole numbers and their opposites are joined together the set of **integers** is created.

The set of integers are indicated in set notation as {…-4, -3, -2, -1, 0, 1, 2, 3, 4…}. These numbers are used to label a number line with the negative numbers located to the left of zero and the positive numbers located to the right of zero. We usually do not write the + sign on the whole numbers or positive integers.

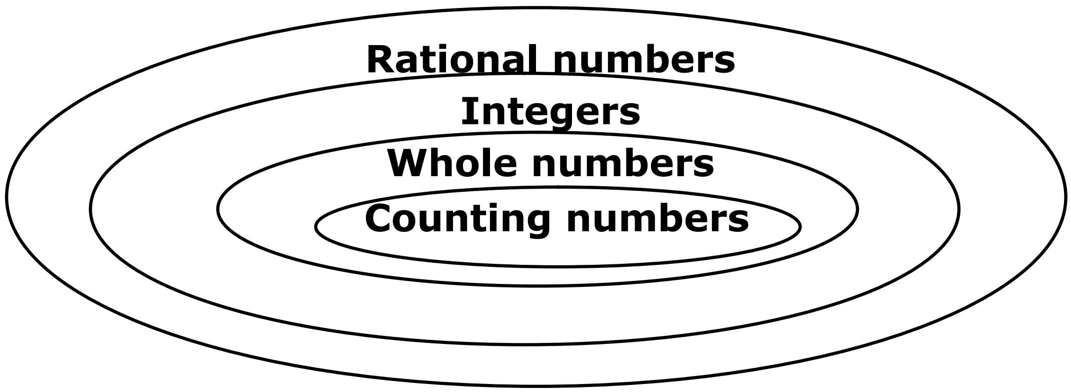


If zero is removed from the set of whole numbers, the set of **natural numbers** or **counting numbers** is created. The natural numbers can be indicated in set notation as $\{1, 2, 3, 4, 5, 6, 7, \ldots\}$.

When a number of the form $\frac{a}{b}$ is created where *a* and *b* are both integers but $b \neq 0$, then the set of **rational numbers** is created. The ratio of 2 to 3 creates $\frac{2}{3}$, so $\frac{2}{3}$ is a rational number. The ratio of 10 to 2 creates $\frac{10}{2}$ or 5 which is a whole number as well as a rational number. Any rational number can be plotted on a number line.

A mixed number like $3\frac{1}{2}$ is a rational number because it can be rewritten as an improper fraction, $\frac{7}{2}$, which is the ratio of two integers.

The relationship between these sets of subsets of the real numbers can be shown with a Venn diagram.



This diagram shows that all counting numbers are whole numbers, all whole numbers are integers, and all integers are rational numbers.

When a set is included completely in another set on the diagram, then all members of the smaller set are also members of the larger set.

Some decimals are rational numbers and some decimals are not rational numbers. If the decimal terminates (ends) OR it has repeating digits, then it is a rational number.

1.25, 1.3, 1.001 are terminating decimals and thus are rational numbers. They can be expressed as a ratio of two integers.

$1.25 = \frac{125}{100} = \frac{5}{4}$ $1.3 = \frac{13}{10}$ $1.001 = \frac{1001}{1000}$

$0.\overline{3}$ and $0.\overline{6}$ are two of the most commonly used repeating decimals.

$0.\overline{3} = \frac{1}{3}$ $0.\overline{6} = \frac{2}{3}$

Some other commonly used repeating decimals are $0.\overline{1}$, $0.\overline{2}$, $0.\overline{4}$, etc. These are the decimals to represent the ratios $\frac{1}{9}$, $\frac{2}{9}$, $\frac{4}{9}$, etc.

2.12345…. and 0.545445444…. are examples of decimals that are not rational numbers. They do not terminate nor do they have repeating digits.

Example: Classify each number by naming the set(s) that it belongs to.

$\frac{1}{4}$ Rational number It is ratio of integers 1 and 4.

-0.15 Rational number It is the ratio of integers -15 and 100

43 Counting number, whole number, integer, and rational number.

-13 Integer, rational number

$0.\overline{7}$ Rational number It is the ratio of integers 7 and 9

Problem-Solving Model

Step	Description of Step
1	**Analyze the given information.** • Summarize the problem in your own words. • Describe the main idea of the problem. • Identify information needed to solve the problem.
2	**Formulate a plan or strategy**. • Draw a picture or diagram. • Guess and check. • Find a pattern. • Act it out. • Create or use a chart or table. • Work a simpler problem. • Work backwards. • Make an organized list. • Use logical reasoning. • Brainstorm. • Write a number sentence or an equation
3	**Determine a solution.** • Estimate the solution to the problem. • Solve the problem.
4	**Justify the solution.** • Explain why your solution solves the problem.
5	**Evaluate the process and the reasonableness of your solution.** • Make sure the solution matches the problem. • Solve the problem in a different way.

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Problem-Solving Questions

Directions:
- **Work with a partner.**
- **Write your answers on notebook paper.**
- **Answer questions 1-3.**
- **Complete the solution to the problem(s).**
- **Answer questions 4-10.**

1. What is the main idea of this problem?
2. What are the supporting details in this problem?
3. What skills, concepts, and understanding of math vocabulary are needed to be able to answer this problem?
4. Did this problem involve mathematics arising in everyday life, society, or the work place?
5. What is a good problem solving strategy for this problem?
6. Can you explain how you used any math tools, mental math, estimation, or number sense to solve this problem?
7. Did this problem involve using multiple representations (symbols, diagrams, graphs, math language)?
8. Did you use any relationships to solve this problem?
9. How can you justify your solution to the problem?
10. How can you check for reasonableness of your solution to this problem?

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Problem-Solving 1

Problem 1: Which of the following statements are true? Use T or NT.

______1. All integers are whole numbers

______2. Any rational number can be expressed as the ratio of two integers.

______3. Some decimal numbers are not rational numbers.

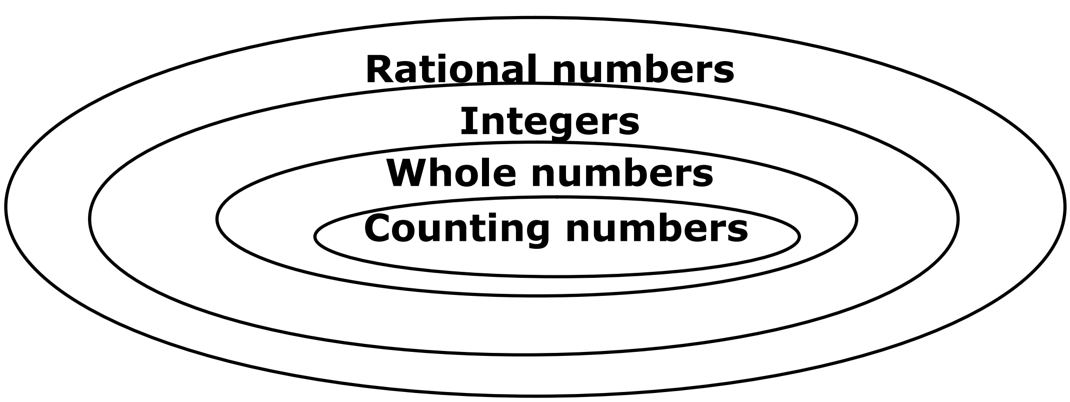
______4. All integers are also rational numbers.

______5. The set $\{8,\ 8.5,\ 10,\ -23\}$ are all rational numbers.

______6. The set $\{-3,\ 19,\ 20,\ 0,-1\}$ are all integers.

For any statement you listed as NT, explain your reasoning.

Problem 2: Place -6, 0, 3.5, $\frac{12}{4}$, $-3\frac{1}{2}$ and $0.\overline{45}$ in the appropriate place on the Venn diagram.



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Understanding Number Sets in Real-World Situations

Numbers used in real-world situations can be whole numbers, integers, and rational numbers. When identifying the set of numbers that could be used in a particular situation, select the one that gives the most precise set. For example, if counting numbers is the set used to describe a situation, you could say whole numbers, integers, and rational numbers. By using the most specific set, counting numbers, it is understood that the order sets would work also.

Example: The number of dimes in a person's pocket

Whole numbers. This set best describes the number of dimes because the person may have 0, 1, 2, 3, … dimes in his pocket.

Example: The lengths of ribbon on 5 spools of ribbon

Positive rational numbers. This set best describes the lengths of ribbon on the spools because the lengths can be numbers like 3.5, $\frac{1}{2}$, 3, etc. Measurements must be positive numbers.

There are subsets of various sets of numbers that are described by a characteristic. For example, the whole numbers divisible by 6 are a subset of the whole numbers. This set would be 6, 12, 18, 24, 30, etc. This set could also be called the multiples of 6.

Some other subsets are even numbers, odd numbers, composite numbers, prime numbers, etc. All of these subsets we have studied in prior grades.

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Problem-Solving 2

Problem 1: Identify the set of numbers that best describe the situations below.

- Numbers used in a phone number

____

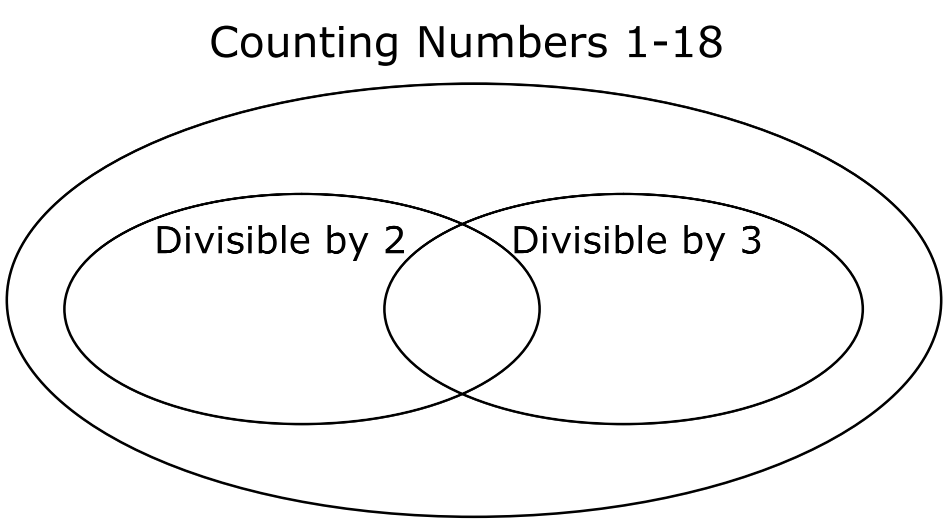
- Golf scores on a leaderboard

____

- Total cost of grocery store purchases

____

Problem 2: Place the counting numbers 1-18 on the Venn diagram below.



Six Weeks 3
Lesson 4

Determining Theoretical and Experimental Probabilities Using Data

Recall from an earlier lesson, the probability of an event is the ratio of the number of favorable outcomes to the number of possible outcomes.

$$P(\text{event}) = \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}}$$

There are two types of probabilities. One probability is called the theoretical probability. The **theoretical probability** ratio is still:

$$\frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}}.$$

For example, if you toss a coin, there are 2 possible outcomes, a tail or a head. The theoretical probability of tossing a head is $\frac{1}{2}$. The theoretical probability of tossing a tail is also $\frac{1}{2}$.

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The other type of probability is called the **experimental probability**. The experimental probability ratio is still:

$$\frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}}.$$

The data used in the ratio is based on an experiment of trials. For example, if you toss a coin 50 times, you could get 25 heads and 25 tails which would match the theoretical probability. However, it is more likely you would get data such as 27 tails and 23 heads. If this were the data you recorded for tossing a penny 50 times, the experimental probability of tossing a head would NOT be $\frac{1}{2}$. It would $\frac{\text{number of heads}}{\text{number of tosses}}$ or $\frac{23}{50}$, which is less than $\frac{1}{2}$.

The more possible outcomes you have, the more likely the experimental probability will be closer to the theoretical probability. If you tossed a coin 2,000 times, you would expect the number of heads and the number of tails to be close to 1,000 each or $\frac{1}{2}$ of the tosses.

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Since a probability is a ratio of two values, it can be expressed as a fraction, decimal, or percent. If the probability is $\frac{1}{2}$, then it can be written also as 0.5 or 50%.

Example: A bag contains 5 red tiles, 6 blue tiles, and 4 yellow tiles. Benny randomly selects a tile from the bag. What is the theoretical probability he will select a red tile? a yellow tile? a blue tile?

$$P(r) = \frac{\text{number of red tiles}}{\text{number of tiles}} = \frac{5}{15} = \frac{1}{3}$$

The theoretical probability of randomly drawing a red tile is $\frac{1}{3}$ or $33\frac{1}{3}\%$.

$$P(y) = \frac{\text{number of yellow tiles}}{\text{number of tiles}} = \frac{4}{15}$$

The theoretical probability of randomly drawing a yellow tile is $\frac{4}{15}$ or $26\frac{2}{3}\%$.

$$P(b) = \frac{\text{number of blue tiles}}{\text{number of tiles}} = \frac{6}{15} = \frac{2}{5}$$

The theoretical probability of randomly drawing a blue tile is $\frac{2}{5}$ or 40%.

Example: A bag contains 5 red tiles, 6 blue tiles, and 4 yellow tiles. Benny randomly selects a tile from the bag. Benny draws a tile from the bag, records it color, and returns the tile to the bag before drawing another tile. He does this 40 times. The results of his experiment are recorded in the table below.

Color	red	blue	yellow
No. of Draws	12	18	10

Based on Benny's data from his experiment, what is the experimental probability the next tile he draws from the bag will be red?

$$P(r) = \frac{\text{number of red draws}}{\text{number of draws}} = \frac{12}{40} = \frac{3}{10} \text{ or } 30\%$$

Based on Benny's data from his experiment, what is the experimental probability the next tile he draws from the bag will be yellow?

$$P(y) = \frac{\text{number of yellow draws}}{\text{number of draws}} = \frac{10}{40} = \frac{1}{4} \text{ or } 25\%$$

Based on Benny's data from his experiment, what is the experimental probability the next tile he draws from the bag will be blue?

$$P(b) = \frac{\text{number of blue draws}}{\text{number of draws}} = \frac{18}{40} = \frac{9}{20} \text{ or } 45\%$$

Example: Using the two examples above, compare the theoretical probability to the experimental probability of drawing each color tile.

RED:

The theoretical probability of drawing a red is $\frac{1}{3}$ or $33\frac{1}{3}\%$. The experimental probability of drawing a red is $\frac{3}{10}$ or 30%. The theoretical probability is slightly larger than the experimental probability.

YELLOW:
The theoretical probability of drawing a yellow is $\frac{4}{15}$ or $26\frac{2}{3}\%$. The experimental probability of drawing a yellow is $\frac{1}{4}$ or 25%. The theoretical probability is slightly larger than the experimental probability.

BLUE: The theoretical probability of drawing a blue is $\frac{2}{5}$ or 40%. The experimental probability of drawing a blue is $\frac{9}{20}$ or 45%. The experimental probability is slightly larger than the theoretical probability.

The sum of the three theoretical probabilities and the sum of the three experimental probabilities must each be 1 or 100%. Use it as a check to make sure you have not miscalculated.

Simple events are when there is one event. The event can be tossing a coin, rolling a number cube, drawing a card, spinning a spinner, etc.

Compound events are when you have more than one event occurring. The events could be tossing a coin and spinning a spinner, tossing a coin and rolling a number cube, drawing a card and spinning a spinner, spinning 2 different spinners, etc.

Compound events can be events that are independent events or dependent events.

Independent events are events that the results of one event do NOT affect the results of the other event. An example of independent events is tossing a coin and spinning a spinner.

If the results of one event do affect the results of the second event, then they are **dependent events**. An example of dependent events is drawing 2 tiles from a bag, one at a time, and NOT replacing the first tile before drawing the second tile. The first draw affects the number of tiles in the bag for the second draw.

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Where there are two independent events, the probability of certain events occurring is the product of the probability of each event occurring.
P(A and B)= P(A) $\times$ P(B)

When there are two dependent events, the probability of certain events occurring is the probability of the first event times the probability of second event occurring given the occurrence of the first event.

This is written P(A and B)= P(A) $\times$ P(B/A)

Example: A bag contains 5 red marbles and 10 blue marbles. You are to select a marble, record its color, replace the marble in the bag, and then draw a second marble. What is the probability you will draw 2 marbles that are red?

The P(r) = $\frac{5}{15} = \frac{1}{3}$ for the first draw. The P(r) = $\frac{5}{15} = \frac{1}{3}$ for the second draw.

The P(r and r) = $\frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$.

These were independent events.

Example: A bag contains 5 red marbles and 10 blue marbles. You are to select a marble, record its color, do NOT replace the marble in the bag, and then draw a second marble. What is the probability you will draw 2 marbles that are red?

The P(r) = $\frac{5}{15} = \frac{1}{3}$ for the first draw. The P(r) = $\frac{4}{14} = \frac{2}{7}$ for the second draw. (1 red has been drawn and NOT replaced so there are only 4 red marbles now and there are only 14 marbles)

The P(r and r) = $\frac{1}{3} \times \frac{2}{7} = \frac{2}{21}$.

These were dependent events.

The probabilities of drawing 2 red marbles are not the same for the two examples. Replacing the marble back in the bag before drawing the second marble makes the events independent. Which situation had the greater probability of occurring?

Problem-Solving 1

Problem 1: Eight number cards are placed face down on a table. The numbers on the cards are 1, 3, 5, 6, 8, 10, 11, and 15. Barbara randomly selects a card and turns it over. What is the probability she selected:

a) the 11 card

b) a card with an even number

c) a card with an odd number

d) a card with a number with 2 digits

e) a card with a number that is a factor of 15

Problem 2: Sarah rolled a 1-6 number cube 54 times. The results of her experiment are shown in the table below.

Number	1	2	3	4	5	6
Frequency	6	9	10	9	10	10

- What is the experimental probability of rolling a 3 the next roll?

- What is the experimental probability of rolling a 2 the next roll?

Determining Theoretical Probability and Experimental Probability using Sample Spaces

A **sample space** of an event is a set of all the possible outcomes of the event. The set can be a list, a tree diagram, or a table.

A sample space for tossing a coin is {heads, tails}. A sample space for rolling a 1-6 number cube is the list: 1, 2, 3, 4, 5, and 6.

To determine the probability of an event using a sample space, use the same ratio

$$\frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}}.$$

Look at the sample space and count the number of favorable outcomes for the numerator of the ratio. Count the total number of entries in the set for the denominator of the ratio.

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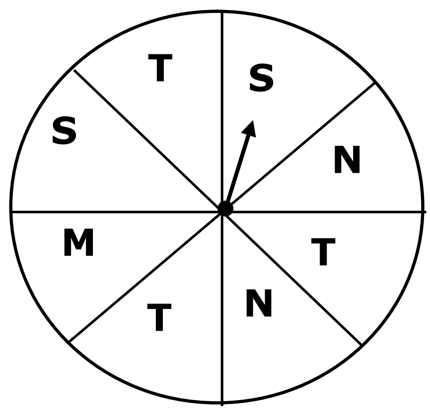
Example: What is the probability of rolling a 6 on a 1-6 number cube?

A sample space for rolling a number cube is {1, 2, 3, 4, 5, 6}. The number of favorable outcomes (6) is 1. The number of entries in the set is 6. The ratio that represents the probability is $\frac{1}{6}$.

Example: If you spin the spinner below, what is the probability you will spin a T?



A sample space for the spinner is {S, T, S, N, T, N, T, M}. The probability or spinning a T is

$\frac{\text{number of Ts}}{\text{number of outcomes}}$. $P(T) = \frac{3}{8}$

Example: You are rolling a number cube and tossing a coin. What is the probability you will roll a 4 and toss a tails?

A sample space for the number cube is {1, 2, 3, 4, 5, 6}

A sample space for tossing a coin is {heads, tails}

$P(4) = \frac{1}{6}$ $P(\text{tails}) = \frac{1}{2}$

$P(4 \text{ and tails}) = \frac{1}{6} \times \frac{1}{2} = \frac{1}{12}$

A sample space for both events could be the list: 1/tails; 1/heads; 2/tails; 2/heads; 3/tails; 3/heads; 4/tails; 4/heads; 5/tails; 5/heads; 6/tails; 6/heads

There are 12 items in the list and 1 in the list is 4/tails. The probability would be $\frac{1}{12}$.

Example: A dessert store kept a record of the number of slices of apple pie they sold one day last week. They also recorded the choice of topping. The table shows the sample space of the apple pie slices sold that day.

Topping	Whipped Cream	Ice Cream	No Topping
Number Served	32	25	43

What is the probability the next slice of apple pie ordered will have a whipped cream topping?

Total the number of slices served. 32 + 25+ 43 = 100. P(WC) = $\frac{32}{100}$ or 32%.

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Problem-Solving 2

Problem 1:

- Give a sample space for spinning a spinner that has 5 equal spaces numbered 1,2,3,4, and 5.

- What is the probability you will spin a 3?

- What is the probability you will spin an even number?

Problem 2:

- Create a sample space for spinning the spinner in Problem 1 and tossing a penny.

- What is the probability you will spin a 3 and toss a head?

- What is the probability you will spin a number larger than 2 and toss a tail?

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