TEKSING TOWARD STAAR

MATHEMATICS ®

GRADE 6

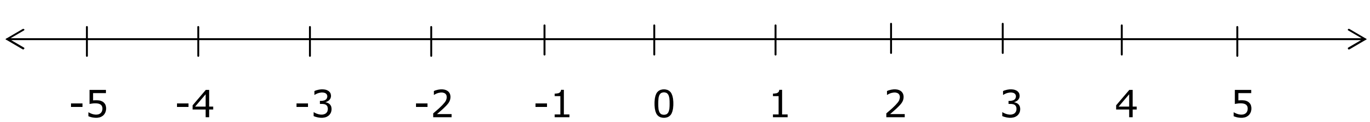
Projections Masters

Six Weeks 1
Lesson 1

Understanding Rational Numbers

A group of items or numbers is called a **set**. A part of that set is called a **subset**. The set of numbers we use in our every day lives is the set of **real numbers.** These are the numbers that are located on a number line. One subset of the real numbers is the set of **whole numbers**. Whole numbers are the numbers 0, 1, 2, 3, 4… Each of these numbers has an opposite 0, -1, -2, -3, -4… When the whole numbers and their opposites are joined together the set of **integers** is created.

The set of integers are indicated in set notation as {…-4, -3, -2, -1, 0, 1, 2, 3, 4…}. These numbers are used to label a number line with the negative numbers located to the left of zero and the positive numbers located to the right of zero. We usually do not write the + sign on the whole numbers.



When a number of the form $\frac{a}{b}$ is created where a and b are both integers but $b \neq 0$, then the set of **rational numbers** is created. The ratio of 2 to 3 creates $\frac{2}{3}$, so $\frac{2}{3}$ is a rational number. The ratio of 10 to 2 creates $\frac{10}{2}$ or 5 which is a whole number as well as a rational number. Any rational number can be plotted on a number line.

Example: Plot $\left\{\frac{2}{3}, \frac{5}{4}, \frac{1}{4}, \frac{7}{2}, 3\right\}$ on the number line.

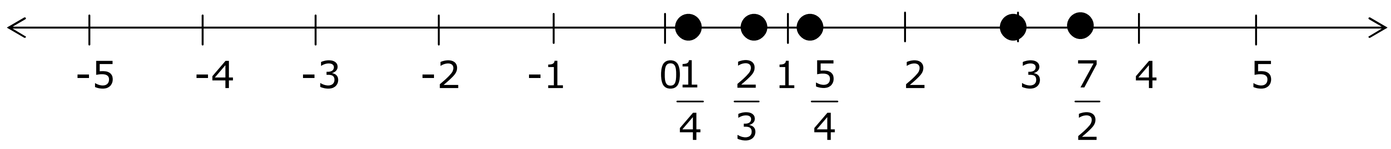
$\frac{1}{4}$ is located one-fourth the distance from 0 to 1.

$\frac{2}{3}$ is located two-thirds the distance from 0 to 1.

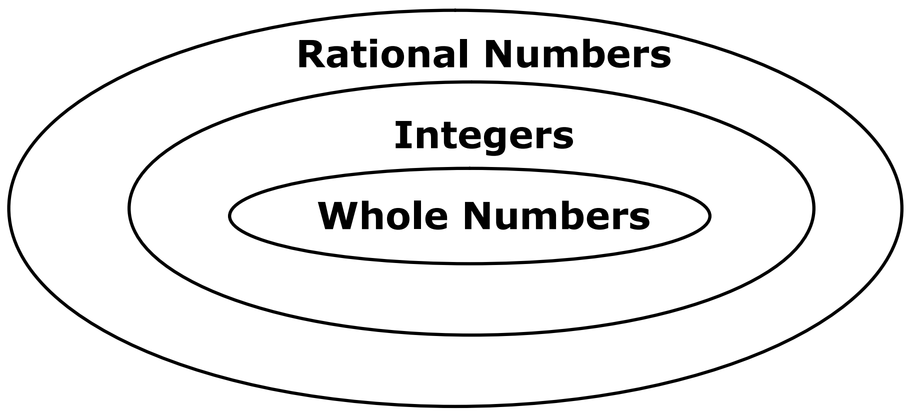
$\frac{5}{4}$ is located one-fourth the distance from 1 to 2.

3 is located at the whole number 3.

$\frac{7}{2}$ is located one-half the distance from 3 to 4.



The relationship between these sets of subsets of the real numbers can be shown with a Venn diagram.



This diagram shows that all whole numbers are integers, and all integers are rational numbers. When a set is included completely in another set on the diagram, then all members of the smaller set are also members of the larger set.

7 is a whole number, integer, and a rational number. -15 is an integer and a rational number. $\frac{15}{7}$ is a rational number only. It is not an integer or whole number.

Some decimals are rational numbers and some decimals are not rational numbers. If the decimal terminates (ends) OR it has repeating digits, then it is a rational number.

1.35, 1.4, 1.075 are terminating decimals and thus are rational numbers. They can be expressed as a ratio of two integers.

$1.35 = \frac{135}{100}$ $1.4 = \frac{14}{10}$ $1.075 = \frac{1075}{1000}$

$0.\overline{3}$ and $0.\overline{6}$ are two of the most commonly used repeating decimals.

$0.\overline{3} = \frac{1}{3}$ $0.\overline{6} = \frac{2}{3}$

2.12345…. and 0.545445444…. are examples of decimals that are not rational numbers. They do not terminate nor do they have repeating digits.

Problem-Solving Model

Step	Description of Step
1	**Analyze the given information.** • Summarize the problem in your own words. • Describe the main idea of the problem. • Identify information needed to solve the problem.
2	**Formulate a plan or strategy**. • Draw a picture or diagram. • Guess and check. • Find a pattern. • Act it out. • Create or use a chart or table. • Work a simpler problem. • Work backwards. • Make an organized list. • Use logical reasoning. • Brainstorm. • Write a number sentence or an equation
3	**Determine a solution.** • Estimate the solution to the problem. • Solve the problem.
4	**Justify the solution.** • Explain why your solution solves the problem.
5	**Evaluate the process and the reasonableness of your solution.** • Make sure the solution matches the problem. • Solve the problem in a different way.

Problem-Solving Questions

Directions:
- **Work with a partner.**
- **Write your answers on notebook paper.**
- **Answer questions 1-3.**
- **Complete the solution to the problem(s).**
- **Answer questions 4-10.**

1. What is the main idea of this problem?

2. What are the supporting details in this problem?

3. What skills, concepts, and understanding of math vocabulary are needed to be able to answer this problem?

4. Did this problem involve mathematics arising in everyday life, society, or the work place?

5. What is a good problem solving strategy for this problem?

6. Can you explain how you used any math tools, mental math, estimation, or number sense to solve this problem?

7. Did this problem involve using multiple representations (symbols, diagrams, graphs, math language)?

8. Did you use any relationships to solve this problem?

9. How can you justify your solution to the problem?

10. How can you check for reasonableness of your solution to this problem?

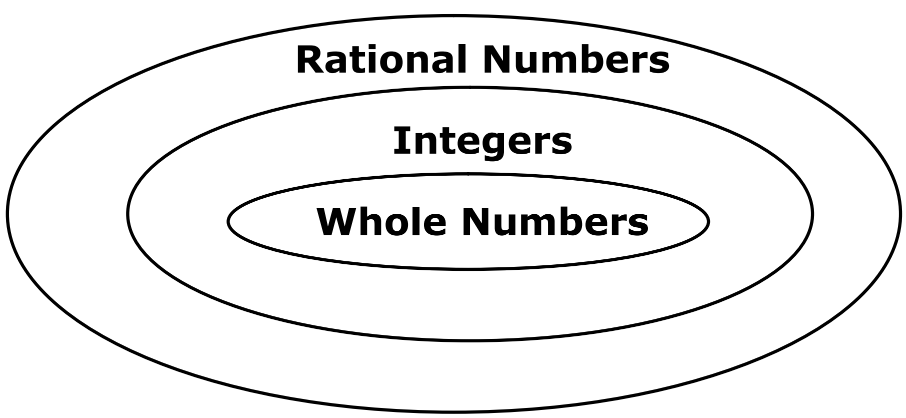
Problem-Solving 1

Problem 1: Which of the following statements are true? Use T or NT.

_____1. All integers are less than 0.
_____2. Any rational number can be expressed as the ratio of two integers.
_____3. All integers are also whole numbers.
_____4. All integers are also rational numbers.
_____5. The set $\{8,\ 8.5,\ 10,\ -23\}$ are all rational numbers.
_____6. The set $\{-3,\ 19,\ 20,\ 0, -1\}$ are all integers.

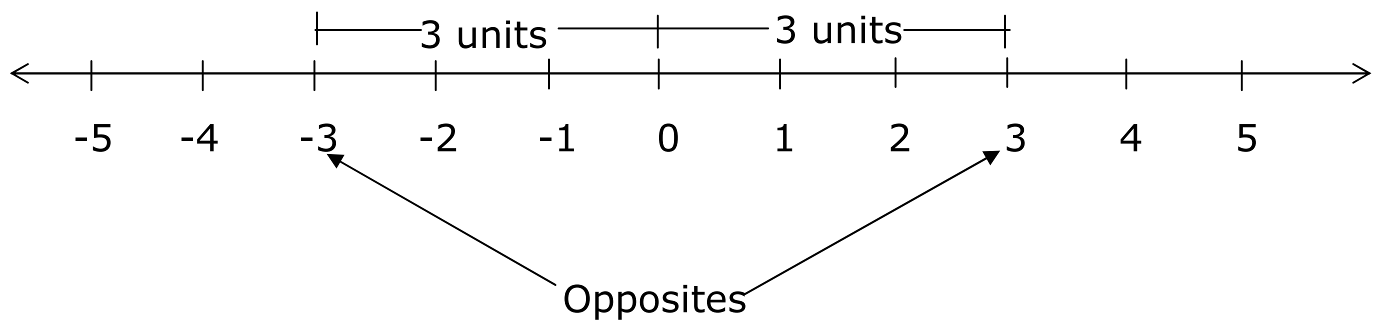
For any statement you listed as NT, explain your reasoning.

Problem 2: Place -6, 0, 3.5, $\frac{12}{4}$, $-3\frac{1}{2}$ and $0.\overline{45}$ in the appropriate place on the Venn diagram.



Understanding Opposites and Absolute Value

The **opposite of a number** is the number on the number line that is the same distance from zero on the number line just on the other side of zero. 3 is 3 units from 0 on the right side. -3 is 3 units from 0 on the left. 3 and -3 are opposites.



When referring to the distance only but not which side of 0, you are referring to the **absolute value** of the number. Thus 3 and -3 both have absolute value 3. The symbol used for absolute value is $|\ |$. $|-5|$ is read "the absolute value of negative 5".

Since -5 is 5 units from 0, the absolute value of negative 5 is 5. We write $|-5| = 5$.

A whole number will be its own absolute value. The absolute value of a negative number will be a positive number, thus its opposite.

Problem-Solving 2

Identify the opposite and absolute value of each of the following numbers.

Number	Opposite	Absolute Value
8		
-14.5		
$3\frac{1}{3}$		
129		
0		
-10		

Which number, or numbers, in the table above have equivalent absolute values and opposite?

Can the absolute value of a number ever be a negative number? Explain

Six Weeks 3
Lesson 2

Understanding Dependent and Independent Quantities

Have you ever heard a comment such as "How long it will take me to get there depends upon the traffic?", or "It will depend upon how much it costs before we can decide if you can have it?"? We deal with some things depending upon other things in our lives every day. In mathematics, we have quantities that depend upon other quantities. They are called **dependent quantities** and the thing they depend upon is called the **independent quantity.**

Some examples of dependent and independent quantities in mathematics are:

1. If you make $15 an hour, your earnings depend upon the number of hours you work.
2. The perimeter of a square depends upon the length of a side of the square.
3. Renting a bike costs $5 an hour. The total rental cost depends upon the number of hours the bike is rented.

When looking at a horizontal table of values, the independent quantity is the first quantity listed and the dependent quantity is the second quantity listed. If it is an *x* and *y* table, then *x* is the first row of values and *y* is the second row of values.

When looking at a vertical table of values, the independent quantity is the first column of values, and the dependent quantity is the second column of values.

Look at the two tables below.

x	3	4	6	9
y	7	6	4	1

x	***y***
2	4
4	8
6	12
8	16

In both tables, *x* is the independent and *y* is the dependent variable. In the first table, *y* is the difference between *x* and 10. In the second table, *y* is twice the value of *x*.

Example: One number is 3 times another number. Complete the table to show possible values of the two numbers.

First Number	1	2	3	4
Second Number				

According to the information given, the second number is three times the first number.
The second number = 3 × first number.

First Number	1	2	3	4
Second Number	$3 \times 1 = 3$	$3 \times 2 = 6$	$3 \times 3 = 9$	$3 \times 4 = 12$

Example: The length of a rectangle is 5 units more than the width. Complete the table to show possible values of the dimensions of the rectangle.

The length depends upon the width. The length is the dependent value and will be the second row of the table. The width is the independent and will be the first row of the table. The equation that shows the relationship is length = width + 5.

Width	1	2	3	4
Length	$1 + 5 = 6$	$2 + 5 = 7$	$3 + 5 = 8$	$4 + 5 = 9$

When looking at a graph, the independent quantity will be the horizontal axis label, and the dependent quantity will be the vertical axis label.

When using a table to create a graph, remember to write the ordered pairs in the correct order, (independent quantity, dependent quantity), and then plot the point belonging to that ordered pair.

Example: Create a table of values and a graph to represent the following situation.

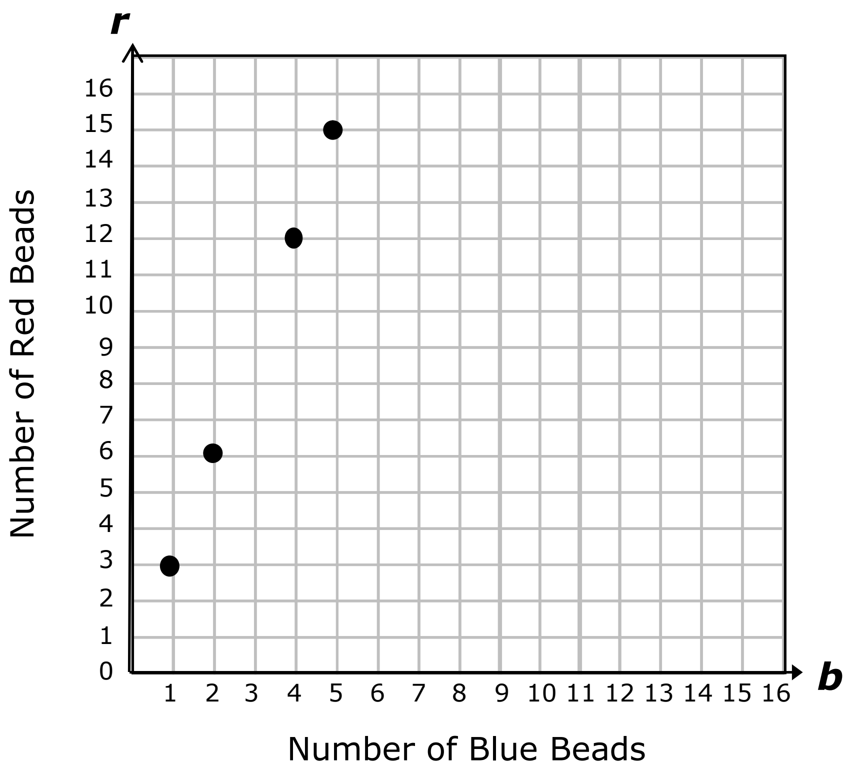
A bag contains red and blue beads. The number of red beads is 3 times the number of blue beads.

The number of red beads depends upon the number of blue beads. The dependent quantity is the number of red beads, *r*. The number of blue beads, *b*, will be the independent quantity.

Table:

b	1	2	4	5
r	$3 \times 1 = 3$	$3 \times 2 = 6$	$3 \times 4 = 12$	$3 \times 5 = 15$

Graph: The ordered pairs from the table are (1, 3), (2, 6), (4, 12), and (5, 15).



Problem-Solving 1

Determine the dependent and independent quantities in the tables or graph below.

Hours	4	5	6	10
Earnings $	60	75	90	150

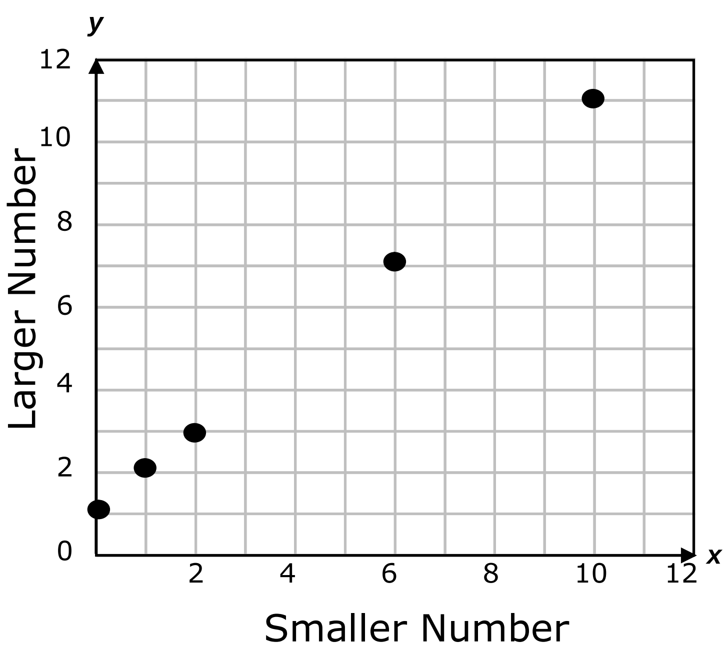
Independent Quantity: __________

Dependent Quantity: ________________

Side of Square (in.)	4	5	6	10
Perimeter (in.)	16	20	24	40

Independent Quantity: __________

Dependent Quantity: ________________



Independent quantity:

Dependent quantity:

Writing an Equation to Represent the Relationship between Independent and Dependent Quantities in a Table

The equations we will use will be of the form $y = ax$ or $y = x + a$. To recognize which of these forms of equations have been used to generate a table, remember what we learned in the last lesson. If this is a situation that is represented by an equation of the form $y = ax$, then the ratio of y to x for each entry in the table will have to be the same value. This value will be the "*a*" in the equation.

Example: Write an equation of the form $y = ax$ or $y = x + a$ that represents the data in the table.

Width (in.), *x*	1	2	3	4
Length (in.), *y*	7	14	21	28

Look at the ratio of y to x in each column of the table.

$$\frac{7}{1} = \frac{14}{2} = \frac{21}{3} = \frac{28}{4} = 7$$

The equation that represents the values in the table is $y = 7x$ or length $= 7 \times$ width.

If the table of values does not have a common ratio, check to see if there is a common difference between *y* and *x* for each set of values. If there is a common difference, then the difference will be the value of "*a*", and the equation will be $y = x + a$

If there is not a common ratio or a common difference, then we will not be able to find the equation yet. We'll leave that kind for grade 7 and grade 8.

Example: Look at the table of values below. Determine the equation that represents the data in the table.

Width (in.), *x*	1	2	3	4
Length (in.), *y*	7	8	9	10

The ratio of *y* to *x* is NOT constant. $\frac{7}{1} \neq \frac{8}{2}$.

Check for a common difference.

$$7 - 1 = 6 \quad 8 - 2 = 6 \quad 9 - 3 = 6 \quad 10 - 4 = 6$$

The common difference is 6. The equation that represents the data in the table is $y = x + 6$ or length = width + 6.

Problem-Solving 2

For each table, write an equation that shows the relationship between *y* and *x*.

x	1	2	3	4
y	2	4	6	8

Equation: ______________________________

x	4	5	20	28
y	12	13	28	36

Equation: ______________________________

x	8	9	20	28
y	2	3	14	22

Equation: ______________________________